APPLICATION OF THE UTILITY THEORY TO THE PROCESS OF DECISION MAKING

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1965

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Submitted to the Department of Chemical and Petroleum Engineering and the Faculty of the Graduate School of The University of Kansas in Partial Fulfillment of the Requirements for the Degree of Master of Science.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to Dr. Kevin R. Jones for his counsel, guidance and support in the preparation of this thesis.

The author also wishes to acknowledge his indebtedness to Dr. Charles F. Weinaug for his overall guidance in the graduate program of Petroleum Management.

A special word of appreciation is due to Dean Wiley S. Mitchell who, as advisor in the School of Business, has been instrumental in offering the Petroleum Management group a most interesting, valuable and challenging course of instruction in Business Administration and has assisted and encouraged the author in this pursuit.

Finally, the author is most grateful to the United States Navy for having granted him the time and invaluable opportunity to pursue these graduate studies in Petroleum Management.

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INTRODUCTION

Purpose

Uncertainty is the father of the art of decision making. As trite as this phrase may seem, it is certainly true today with the rapidly increasing tempo of business and the introduction of more and more complex variables. Truly, "the business executive is by profession a decision maker."[1]

Since the turn of the century, particularly during the period following World War II, an awareness of the ultimate ineffectiveness of purely intuitive conclusions has permeated the research projects in the fields of management. The dominant purpose of this continuing research has been the elucidation of a rational theory for purposive action in situations of risk and uncertainty. Several methods or techniques have been developed to offer a basis upon which to compare and evaluate the expected value or worth of the separate sets of outcomes under consideration. Using these methods, the optimal decision would be the one with the greatest expected value or worth.

To exemplify the methods cited above, consider this simple case. The decision maker has two separate possibilities and he knows the probability associated with each possibility. The sum of these separate possibilities, weighted by the probability that each will occur, is called the expected value of the probability distribution. If the decision maker

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[1]References listed at end.

has an expected monetary return for each of the separate possibilities, he can determine the "weighted average" of each monetary result. Thus he can choose the act with the highest weighted average monetary result. This is called "maximizing expected monetary value."

One vital element has been omitted, or tacitly ignored, in most of these techniques, namely, the value of the outcome for the particular person based upon his particular funds, goals and risk preference. The utility theory has recently become accredited as a means of incorporating the element of risk into decision making.

Various definitions have been ascribed to this theory by the different theorists. A utility is what the researcher takes to be the value of an outcome for a person. Utility is the power to satisfy human wants. It is a measurement of the degree to which satisfaction is obtained or it is a measurement of the degree of pleasure or desire for one object as compared to another. Lastly, it is the actual value of a dollar to the individual when evaluated in terms of his financial status and risk preferences. We shall consider utility as synonymous with preferability; thus the degree of utility will be taken as the degree of preferability. If a person prefers an orange, for example, to an apple, this will indicate that he has a higher utility for an orange than for an apple.

The loss or the gain of a dollar has a far different significance to a business man of limited capital as compared

to a large chain store concern with unlimited resources. In addition, there is a different significance to a man who buys only "blue-chip" stock as compared to the man who deals primarily with speculative stock. The straight maximization of "expected monetary value" theory would not be a satisfactory criterion for a decision maker in either of the above examples. Now if we have a means of assigning a single utility to each outcome, and a single probability to each action-outcome combination, then a decision maker's choice of an action can be predicted by maximizing expected utility. Thus he has included the element of risk into the evaluation.

Inherent in the utility theory approach has been the difficulty in measuring or identifying the utility value. The methods, which have been documented, either involve complicated axioms or lengthy laboratory experimentation. A completely general method of measuring utility, which can be applied in all situations, has yet to be developed.

The purpose of the work reported in this thesis was to investigate ways to make utility calculations by digital computer possible. In order to do so it was necessary to devise a basic mathematical function which would work for all, or at least a majority, of the empirical utility curves. This accomplishment could prove of two-fold usefulness. First, it would make available to a decision maker an easily administered procedure to generate data for individual utility curves. Secondly, it would provide a mechanized way to compute the utility-dollar values, to determine the utility function and to plot the resultant curve.

It is hoped that this thesis can lend support to utility theory as an aid to decision making and as a rule for guiding judicious behavior in the face of uncertainty. If so, possibly it will find general acceptance as an economic evaluation technique in the future.

Contemporary Yardsticks

Payout time is the time required for the cumulative net earnings from an investment to equal the investment. Probably this technique is the criterion by which the majority of all properties are evaluated even though it is inadequate in many respects. The primary weakness of this procedure is that it considers only the time of recovery and does not take into consideration the expected total earnings of the investment which depend upon the rate of earnings and the life of the investment. Payout time has the advantage of being simple to understand and, therefore, is more widely known by decision makers.

Profit-to-investment is the ratio of the total return from the investment to the amount of the investment. This procedure considers the total return and thus projects a ready balance sheet estimate for the project, but does not reflect the rate of return.

The net present value refers to all future net revenues before taxes, discounted back to the date of evaluation. This procedure incorporates another valuable dimension into the analysis, namely, the present value of annuity earnings in the future. In the long form approach, the net revenue

for each year is discounted back to the present. Although this step-by-step procedure is more time consuming, it offers the advantage of giving a detailed picture of the future earnings. The short-cut approach utilizes the basic expression for exponential (constant percentage) decline.

A derivative of the net present value method is the profitability index which is the present value of earnings divided by the investment. The earnings are discounted over the period of the annuities and calculated on the basis of the present worth values of such annuities. Some disadvantages of this method, which probably have restricted its popularity, are: (1) the method requires that an earning rate be established before calculations are made, and (2) since the index is merely an abstract number, it is not as readily appreciated by management and stockholders.

The rate of return is essentially the interest rate for which the "present worth" of the future income is equal to the investment. This rate is determined from the formula:

$$P = C\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

where: i = interest rate for a given period

n = number of interest periods

P = principal sum invested at the present effective date of the valuation

C = one of the series of equal payments made at the end of each interest period.

The present value techniques have gained recognition as better criteria than the payout or profit-to-investment because they include the effect of time on the value of money. Unfortunately, they do not include the element of risk in the comparison and evaluation processes. Whether or not the decision maker consciously includes it in his evaluation, this element is ever present. Any technique which will introduce risk into the evaluation process will be a step toward providing a better standard for the decision maker.

The Problem

Modern business terminology includes qualitative and quantitative outcomes resulting from decisions made under certainty, made under risk, and made under uncertainty. Although definitions of these terms are generally consistent, the author believes that more clarity will result by specifying the definitions as used in this paper.

If each action is known to lead invariably to a specific outcome, we have certainty. If each action leads to one of a set of possible outcomes and each outcome occurs with a known probability, we have risk. If each action that may be chosen is identified with a distribution of potential outcomes rather than a unique outcome, we have uncertainty. If the outcome is one which, following the choice of a course of action, is either obtained or not obtained, we have a qualitative outcome. If the outcome is one which is or is not obtained in various degrees but the extent to which such

an outcome is obtained is potentially measurable, we have a quantitative outcome.

The simplest type of problem situation is one which would involve only two possible outcomes, both qualitatively defined, and two courses of action. It is necessary to know the relative values of these outcomes to the decision maker in order for him to determine which of these two courses of action would be better. Recognition of the necessity for determining the relative importance of outcomes to decision makers has given rise to the value or utility theory.

As a generalization, it may be said that insofar as it can be measured quantitatively, the over-all objective of a business adventure is to maximize the profit on the investment, consistent with maintaining a sound financial position.[2] One of the most prominent "maximization" analysis of rationality is the hypothesis that the decision maker should maximize expected utility or value with respect to his beliefs concerning the facts of the situation. To perform this maximization, he needs to have a subjective probability function which measures his degree of belief and a utility function which will measure the relative value to him of the various possible outcomes of his actions or decisions.

Thus the problem is to derive the subjective probability function that will measure these relative values for the operator so that he can make the optimal decision which, consistent with our definitions, will maximize the mathematical expectation of value or utility.

The Hypothetical Model

Uncertainty in management decisions generally can be contributed to at least two major sources: imperfect foresight and human inability to solve complex problems containing a host of variables even when an optimum is definable. Under our definition of uncertainty, each action which may be chosen is identified with a distribution of potential outcomes rather than with a unique one. To help define the boundaries of the potential outcomes, many companies have made use of models. This has permitted the substitution of empirical methods for the former purely intuitive or rational grounds for selecting a criterion.

A model, by the simplest definition, is a representation of reality which attempts to explain the behavior of some aspect of it. Because a model is an explicit representation of reality it is always less complex than the reality itself, but it has to be sufficiently complete to approximate those aspects of reality which are being investigated.[3] The model is constructed so that the parameters represent as many input variables as possible. By varying one or more parameters at a time while keeping the remainder constant, the limits to the distribution of potential outcomes can be defined. In the past, time and human error have been the major deterrents to practical use of any necessarily complex model. With the advent of the digital computer, management now has a convenient means of obtaining timely and accurate results even with the most complex models.

Grayson[4] proposed that the decision maker assign a subjective, or "utility," value to potential dollar consequences. To accomplish this, the operator must first construct a utility function or curve for himself. Utilizing this function or curve, he can convert dollar consequences of act-events into units of measurements, called utiles, which reflect what the dollar really means to him. He then assigns probabilities of occurrence to each event. The weighted average consequence of each act is found by weighting the utiles by the probability. He is now able to select the act which will give the largest weighted average consequence.

We shall use the Grayson thesis to a large extent in setting up his model. The model will consist of a short questionnaire and an associated computer program. The questionnaire can easily be administered to management personnel or even to prospective employees. The probabilities (answers) from the questionnaire will be converted to utiles by the computer program. Then the computer will attempt to derive an equation to satisfy the dollar-utile data points. Using the derived equation, the computer will plot the utility function for the participant. From this point, the uses of the plotted utility function will be limited only by the practical use desired by the person or firm.

Plan of the Thesis

The general outline for the thesis will be the development and detailed description of the model which has been introduced in this chapter.

The evolution of the utility theory will be traced from the early Bernoulli "diminishing utility curve" to the current period in Chapter II. A resumé of the different theories of measurability of the utility value and references to the major contributions in this field will also be found in this chapter.

In Chapter III we develop the mathematical equation which will be used in processing the data used in the research. The various methods of regression analysis which form the basis for the computer programs will be briefly explained.

The development of a questionnaire, which is our suggested contribution to the problem of measurability of the utility value, will be found in Chapter IV.

A method of simultaneously incorporating the data from Part II with those from Part I is developed in Chapter V.

The practical usefulness of the questionnaire was tested by 200 volunteer students from the School of Business, University of Kansas. The results of this test form the basis of Chapter VI.

Chapter II

THE DEVELOPMENT OF THE UTILITY THEORY

Historical Perspective

The concept of Utility Theory is not a recent innovation. Kauder[5] credits Kraus[6] with discovering some early discussion of the law of diminishing utility in Aristotle's works.[7] In 1730, the mathematician, Daniel Bernoulli, dealing primarily with the games of chance, isolated the theory of marginal value as applied specifically to the value of money.[8] He theorized that the value of the last item received of a commodity is the marginal utility. His "diminishing utility curve" equation:

$$u_x = a \log (x) + b$$

is equivalent to the basic equation for the research of this paper.

Adam Smith made the distinction between value in use and value in exchange in 1776.[9] Jeremy Bentham, through his calculus of values, introduced the concept of utility as a numerical magnitude. He also utilized the principle in his suggestion for the measurement of quantities of pleasure and pain,[10] and for the creation of the following corollaries: (1) gambling is utility-decreasing and, (2) insurance is utility-increasing.[11]

It was left to Heinrick Gossen to publish the fundamental principle of marginal utility theory. He concluded that: "a person maximizes his utility when he distributes his available

money among the various goods so that he obtains the same amount of satisfaction from the last unit of money spent upon each commodity."[12]

Jevons was probably the first person to introduce utility as a positive and integral part of economic philosophy. In the 1860's he attempted to reconstruct political economy with his speculative calculus of pleasure and pain. This concept later was termed by economic theorists as the preference approach to utility. He said "value (was) to be established on the basis of labor and the problem of rent, wages, interest, etc., (and was) to be solved as mathematical functions."[13] La Nauze stated that Jevons' works mark the "beginning of economics, as distinct from political economy. (His) utility theory made economics a 'serious' subject, a technical subject."[14]

Walras,[15] Marshall[16] and Edgeworth[17] concluded that utility was a quantity which could be measurable if the operator had sufficient facts. Pareto definitely abandoned this approach; he returned to the Jevons concept of a scale of preferences.[18]

The first careful examination of the measurability of the utility function and its relevance to demand theory was made by Fisher in 1892. He resolved the measurability problem for the case in which the marginal utilities of the various quantities are independent of one another.[19]

In summation, the evolution and development of the utility theory has had periods of virtually universal appeal followed by lengthy periods of dormancy.[20] Following

Bernoulli's mathematical derivation of the "diminishing utility curve" in 1730, little was documented, if any work was actually done, until Gossen's publication sparked the pioneering efforts of Jevons, Marshall and Walras. Fisher became the first American to seriously advocate and demonstrate the applications of the theory. Even with the advent of modern methods of publication and distribution, another dormant period existed until 1926.

Contemporary Perspective

Ragnar Frisch could well be considered the father of the modern surge of research into the application of the theory. He introduced his axiomatic method in his 1926 article under the heading of "axioms of the second kind."[21] This method has proved itself of great value particularly in geometry and probability. While many developments have taken place in axiomatizing economic theory,[22,23,24,25] undoubtedly the greatest impetus of recent years has come from the work of von Neumann and Morgenstern[26] and Marschak.[27]

Although the comparison of adjacent preferences appears to have its first explicit recognition by Pareto,[28] the modern introduction of this concept appears in Frisch's article. In 1934, after the appearance of the Hicks-Allen contribution[29] the question of adjacent preferences was revived by Lange[30] followed by Samuelson,[31] Weldon,[32] Suppes and Winet,[33] and von Neumann and Morgenstern.[34]

In 1954, several authors presented separate articles on an entirely new approach. Hausner[35] and Thrall[36] are credited

with establishing the existence of a lexicographic utility function from their investigation of the non-Archimedan behavior under uncertainty. Debreu made an analogous investigation in terms of the classical commodity space,[37] while Georgescu-Roegens' work added significance to the concept.[38] Chipman defined utility, in its most general terms, as "a lexicographic ordering, represented by a finite or infinite dimensional vector with real components, unique only up to an isotone (order-preserving) homogeneous transformation; and these vectors (or 'lexical numbers') are ordered lexicographically like decimal numbers or words in a dictionary. Less generally still the components of the lexical numbers may be additive (in the Bernoulli case) or multiplicative (in the Frisch case); at any rate, unique up to a homogeneous transformation which preserves group operations."[39]

Measurability

Because there have been so many thorough and learned dissertations in the last decade on the various ideologies and definitions appertaining to utility theory, it would be impractical and useless to attempt to condense the major contributions into this thesis. Accordingly, detailed explanations of the various theories will be omitted; only references to selected measurement theories will be included.

The concept of a measurable utility, i.e., of a real-valued function appropriately linear with respect to probability distribution, measuring an individual's preference ratings, originated with Bernoulli. A completely rigorous

formulation and treatment of the existence of such a utility, on the basis of a well-defined set of axioms or postulates, however, was unknown until Fisher's work appeared in 1892. This was followed by von Neumann and Morgenstern in their Theory of Games.[40] Marschak developed a set of axioms for the case of a finite number of sure prospects.[41] Rubin extended the Marschak system to the case of an infinite number of such prospects.[42] For a description of von Neumann and Morgenstern's axioms and three more recent methodologies with their associated axioms, see Ackoff.[43]

Hick and Allen introduced the ordinate theory as opposed to the then existing cardinal ideas.[44] Further studies deriving the existence of an ordinate utility function from postulates about preferences have been made by Wold[45] and by Debreu.[46] For lengthy discussions on cardinal verses ordinal utility see Little,[47] von Neumann and Morgenstern[48] and Robertson.[49] The major difference between Hicks-Allen utility and classic marginal utility concerns the assumption of the orderability of marginal utility.

Armstrong introduced marginal preference[50] while von Neumann and Morgenstern speaks of risk index or utility index[51] as a measure. Davidson and Marschak,[52] Luce[53] and Papandreau[54] studied stochastic choice behavior as a means of measurement.

A logical approach to a discussion of utility is shown by Kennedy[55] who says, "Thus we shall hold that utility is a quantity, i.e., has magnitude; that it is indivisible; and that two utilities when added together do not yield another

utility...we shall not attempt to decide whether utility should properly be regarded as pleasure, as happiness, as satisfaction, as intensity of desire or as liking."

Fisher postulates that by the use of his axioms, "utility as defined...does not involve the economist in controversy as to the laws of the subjective states of pleasure and pain, the influence of their anticipation as connected with their probabilities, the vexed questions whether they differ in quality as well as in intensity and duration, whether duty can or cannot exist as a motive independently of pleasure,.."[56]

Modern Applications of the Utility Theory

An exhaustive resumé of the various applications to which utility theory has found acceptance today would be prohibitive in this thesis. A few examples will be cited below to portray the magnitude of the spectrum of applications which reflect the versatility of the theory.

Since World War II there has been a tremendous increase of interest in the use of scientific methods in solving management problems, under such labels as "operations research", "management science", etc. Churchman,[57] Ackoff,[58] Miller and Starr,[59] Thomas and Deemer,[60] to mention only a few, have included utility theory as a possible scientific method for use in decision making.

Grayson attempted to simplify drilling decisions by oil companies by use of utility theory.[61] Kaufman has extended Grayson's work in this field.[62]

Koopmans asserts that "it is shown that simple postulates about the utility function of a consumption program for an

infinite future logically imply impatience at least for certain broad classes of programs."[63]

Tinbergen attempted, with the help of measured economic concepts, to indicate the rate of savings, as a function of time, which maximized utility over tine.[64]

Samuelson used value assumptions to help him move straight from the utility of the individual to the welfare of society.[65] Piguo and Little made use of inter-personal utility comparisons to take them from the utility of the individual to the utility of the community and only then introduce value assumptions to bring them finally to economic welfare.[66]

The UCLA Design Research Program has experimented with the application of the utility function in devising a methodology for defining and applying consistent, realistic value quantities useful in decision-making at all levels of the design process.[67]

Although adoption of the utility theory has been extended to an ever increasing number of uses, the process is still in its embryonic state. Utility theory can incorporate personal attitudes toward risk taking into the evaluation of decisions. It should prove to be of substantial assistance in creating consistency in decision making and should provide a basis for attempts to make simple business decisions by computers.

Chapter III

THE MATHEMATICAL EQUATION

Background

As mentioned earlier in Chapter II, Daniel Bernoulli was one of the first to present the general idea of introducing subjective values of dollars into expectation calculations. He rationalized that each equal increment of gain yielded an advantage which was inversely proportional to the individual's wealth, i.e.,

$$dU = K \frac{dx}{x} \tag{1}$$

where dU is the increment of utility resulting from an increment dx of wealth and K is a constant. From this assumption, he projected the total utility as a logarithmic function of wealth in this manner:

$$U_{(x)} = K \log \left(\frac{x}{C}\right) \tag{2}$$

where C is the amount of wealth necessary for existence. Integrating the differential expression, we get his "diminishing marginal utility" equation:

$$U_{(x)} = k \log (x) + C \tag{3}$$

where the constant is determined by the conditions that, when wealth is at the subsistence level C, U = 0.[68] Thus it can be seen that the utility of money was represented as a linear function of the logarithm of the dollar value.

Research done by Kaufmann[69] on the utility function curves derived by Grayson[70] indicated that a logarithmic

function was not the best fit for all cases. In fact he stated that a cubic or a quadratic function fit some of the personal curves better than a logarithmic curve, while in other cases, a quadratic might best fit the negative section (quadrant III) and a cubic might best fit the positive section (quadrant I) of the curve.

Intuitively it would appear that the logarithmic curve would more closely match the curve in Figure (1) than any of the other typical named curves. Without discounting the contribution of Kaufmann, we proceeded on the assumption that in a majority of cases, the logarithmic equation by Bernoulli would give the better fit.

Accordingly, the following equation was used to commence the research:

$$u_{(j)} = a \log ((x_{(j)} + b)/b) \tag{4}$$

The constant which appeared in Bernoulli's equation was incorporated into the basic logarithmic value by the addition of the b in the denominator. This has the effect of forcing the curve to pass through the point (0,0). (With $x = 0$, the logarithm of 1 does equal 0.) There is one requirement; b must always be greater than the largest negative value of $x_{(j)}$ in order to eliminate the possibility of having a logarithm of a negative number. Subject to this requirement we have successfully incorporated the Bernoulli constant within our logarithm expression.

Having established equation (4) as the research equation, we now can look at the methodology for generating the

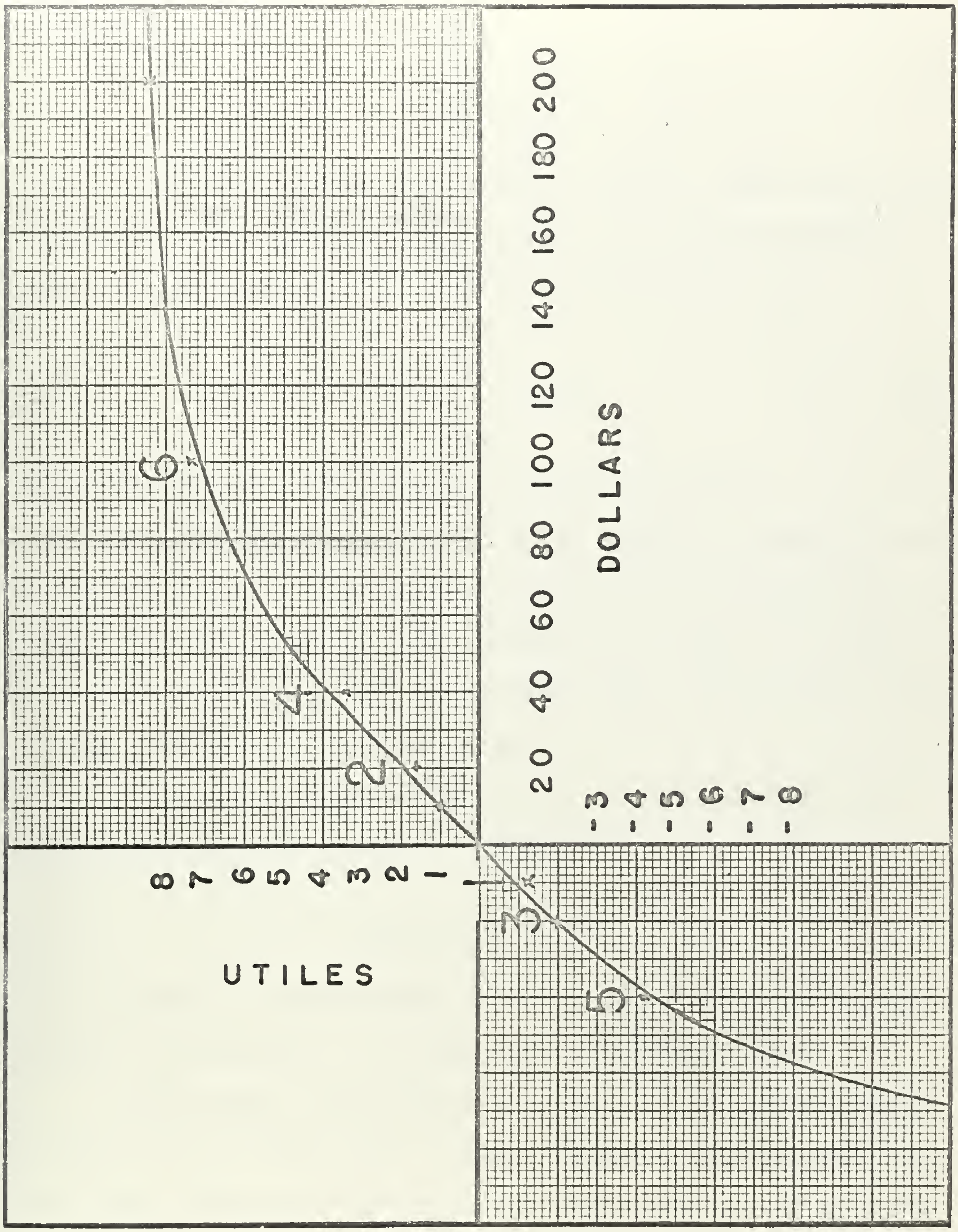

Figure 1

STEP SEQUENCE IN CALCULATING VALUES FOR PART I DATA

data points. The expected value of any distribution of events is:

$$E_u = \sum_{K=1}^{n} P_k V_k \tag{5}$$

where P_k is the probability that the event takes place and V_k is the value of that event. In our case where the value is measured by utility, the expected utility value is:

$$E_u = \sum_{K=1}^{n} P_k U_k \tag{6}$$

where U_k is the utility of the particular event.

In an investigation of utility, a useful concept is that of indifference. A person is said to be indifferent to a proposal if he does not care whether he accepts it or not. If he is indifferent it is axiomatic that the expected value of the proposal is zero for him.

In the case where the proposal has two possible outcomes the expected utility is given by:

$$E_u = P_1 U_1 + P_2 U_2 \quad . \tag{7}$$

If a subject is indifferent to the proposal then $E_u = 0$ and P_1 and P_2 are said to be indifference probabilities.

Indifference preferences have been a recognized theory of measurement within the utility theory. Grayson utilized the indifference probabilities to determine his utility-in-money functions. We have used it in both Parts I and II of the questionnaire where the expected value of the hypothetical

wager was set equal to zero. Thus:

$$U_{winning} P_{winning} - U_{losing} (1 - P_{winning}) = 0 \qquad (8)$$

and $P_{winning}$ is the indifference probability.

<u>General Mathematical Model</u>

The equation of the curve is:

$$u = a \ln [(x + b)/b] \qquad (9)$$

The objective is to determine a curve such that the sum of the squares of the distance of the data points is the least, where e = data points. So let:

$$e_j = u_j - a \ln [(x_j + b)/b] \qquad (10)$$

Now squaring both sides:

$$e_j^2 = [u_j - a \ln ((x_j + b)/b)]^2 \qquad (11)$$

And squaring over all points:

$$E = \sum_{j=1}^{n} (e_j)^2 \qquad (12)$$

We take the partial derivatives of E with respect to a and b and set them equal to zero in order to minimize the total error. The steps in sequential order are listed below.

$$\frac{\partial E}{\partial a} = 2 \sum_{j=1}^{n} (e_j) \frac{\partial e_j}{\partial a} = 0 \qquad (13a)$$

$$\frac{\partial E}{\partial b} = 2 \sum_{j=1}^{n} (e_j)\frac{\partial e_j}{\partial b} = 0 \tag{13b}$$

$$\frac{\partial e_j}{\partial a} = -\ln\left[(x_j + b)/b\right] \tag{14a}$$

$$\frac{\partial e_j}{\partial b} = a\left[1/(x_j + b) - 1/b\right] \tag{14b}$$

Substituting equations (11) and (14) into equation (13) and dividing by -2, we get:

$$\sum_{j=1}^{n} u_j[1/(x_j+b)-1/b]-a \sum_{j=1}^{n} \ln[(x_j+b)/b]\,[1/(x_j+b)-1/b] = 0 \tag{15a}$$

$$\sum_{j=1}^{n} u_j\,[\ln((x_j + b)/b)] - a \sum_{j=1}^{n} [\ln((x_j + b/b)]^2 = 0 \tag{15b}$$

To simplify the equations and to reduce repetitive calculations by the computer, the following terms were collected and identified as follows:

$$Sam_j = \ln\left[(x_j + b)/b\right]$$

$$H_j = \left[1/(x_j + b) - 1/b\right]$$

Now rewriting the partial differentials after introduction of the collective terms, gives:

$$\frac{\partial E}{\partial a} = \sum_{j=1}^{n} u_j H_j - a \sum_{j=1}^{n} Sam_j H_j = 0 \qquad (16a)$$

$$\frac{\partial E}{\partial b} = \sum_{j=1}^{n} u_j Sam_j - a \sum_{j=1}^{n} (Sam_j)^2 = 0 \qquad (16b)$$

In order conveniently to prepare subprograms for the use of the mainline computer program, the following function designations were used:

$$Q = \sum_{j=1}^{n} u_j Sam_j$$

$$R = \sum_{j=1}^{n} Sam_j H_j$$

$$T = \sum_{j=1}^{n} u_j H_j$$

$$W = \sum_{j=1}^{n} (Sam_j)^2$$

Rewriting the partial derivatives after the introduction of the function designations, gives:

$$\frac{\partial E}{\partial a} = T - aR = 0 \qquad (17a)$$

$$\frac{\partial E}{\partial b} = Q - aW = 0 \qquad (17b)$$

and

$$a = T/R \qquad (18)$$

or

$$a = Q/W \qquad (19)$$

so the first derivative of the remaining unknown constant, b, is:

$$FOFB = (T\ W) - (Q\ R). \qquad (20)$$

The Newton-Raphson Method

When the derivative of f(x) is a simple expression and easily found, the real roots of f(x) = 0 can be computed by the Newton-Raphson Method. It is a powerful method of solving an implicit equation. This method was used in the computer program which has been identified in this thesis as the "Original Logarithm Program."

The Twenty Question Method

In this method of iteration, two numbers x_1 and x_2 are found so that the root of the equation lies between them. Since the root lies between the two numbers, the graph of y = f(x) must cross the x-axis between $x = x_1$ and $x = x_2$, and y_1 and y_2 must have opposite signs. Having determined x_1 and x_2, we find a mean value for the two points by:

$$x_m = (x_1 + x_2)/2.$$

We substitute x_m for x in the equation y = f(x) and calculate

a new value for y. For example, let us assume y_1 was negative and y_2 was positive. If y_m is negative the value of x_m is substituted for x_1 and the procedure for finding a new x_m is repeated; if y_m is positive the value of x_m is substituted for x_2 and the procedure for finding x_m is repeated, etc.

Equation (20) of our mathematical model is the general equation with the constant B as the only unknown. To apply this method, FOFB plays the part of y and B plays the part of x. The first steps are to determine two values for B such that FOFB of one (BG) is larger than zero and FOFB of the other (BL) is less than zero. Having established these two limiting points, we used 20 iterations to calculate the correct value of B. If the absolute value of FOFB was less than .00001, the iterations were terminated.

<u>Snolse Method</u>

A computer program designed to solve non-linear simultaneous equations was developed by McLean.[71] This subprogram was used with the mathematical model for this thesis.

Chapter IV

THE QUESTIONNAIRE

Very little has been documented on the exact methods proposed and used in determining the utility function of individuals by other authors or research personnel. Grayson used a table of indifference probabilities on hypothetical drilling propositions.[72] Mosteller and Nagee made extensive laboratory experiments with selected groups of students and National Guardsmen. The experiment consisted of a gaming technique with small amounts of money (from 1 to 100 cents) designed to determine a numerical value of utility for the different sums of money. After obtaining for each individual a utility measure for various amounts of money, they attempted to predict how each individual would choose among a set of uncertain prospects, where the entities were amounts of money with associated probabilities.[73] Although several measurement techniques were mentioned in the previous chapter, none of these theoretical techniques were transcribed into simple usable experiments or questionnaires.

The questionnaire used for the computer program in this thesis is included as Appendix (1). It was designed as a series of questions about investments. To simplify the situation, however, two slightly different games of chance were substituted for investments.

For Part I, the questions concerned a hypothetical game played with a deck of 100 black or red cards. The results of this game depended upon one and only one card being drawn by the hypothetical opponent. Should the opponent draw a

black card, the participant won; should he draw a red card, the participant lost. The answers to the series of entries represented the odds or probabilities requested by the participant. The answers reflected the exact conditions the participant would require in order to make a monetary investment in the amount of the gaming bet of the entry.

For Part II, the game consisted of a series of bets on one toss of a fair coin. In this part, the amount of loss was fixed and the participant was asked to write the amount of win he would require in order to play the game or make an equal monetary investment with the probability of 50/50 of winning.

Although we shall defer the detailed explanation of this point until later in the thesis, it should be noted that the calculations including only Part I data points are much easier than the calculations including Part II data points or a combination of Part I and Part II points. In fact, the calculations of Part II depend upon Part I results for reference. However, initially it was not known which method would create for the participant the more meaningful game and thus give the more accurate personal utility curve so Part II points were included not only for cross reference purposes but also for analysis purposes.

The basic development of the theories underlying the questionnaire can best be explained with reference to Figure (1). This figure shows the order of determining the values of the points from Part I of the questionnaire. It is to be

noted that point (0,0) was assigned as was point (+10,+1). It should also be noted that the next points are determined in this order, -10, +20, -20, or +, -, +, -, etc.

The points (0,0) and (+10,+1) were arbitrarily set. As the utility of zero dollars logically would be zero, the criteria for the establishment of the zeroth point need not be explained further. The dollar value for the first point, to represent a utility of +1, could be any arbitrary amount. However, several factors must be considered in picking this point. The number of entries on the questionnaire should be kept to a minimum; a long questionnaire immediately creates an undesirable effect upon the participant. Yet it is essential to obtain a reasonable number of points in order properly to represent the curve. It is recommended that a number between 12 and 20 points be used. Lastly, to make the questionnaire applicable to persons of varying financial status, the largest monetary entry must be restricted in order to keep its value within the rational understanding of the participant. Thus if a small number like $1 is picked, the questionnaire is either too long or the range of dollar values is too restricted for practical use. If a large number like $50 is picked, the larger monetary limitation can easily be exceeded.

Having established the first two points, the other points are determined from probabilities and dollar values associated with a previously determined dollar-utile point. The equation used in determining the other points was the

indifference equation:

$$U_{win} P_{win} - U_{lose} (1 - P_{win}) = 0 \tag{8}$$

Inherent in this equation is the restriction that the utility of one of the points must be known. Thus each questionnaire entry must contain a dollar value for which the utility value is known. The logical progressive steps for the entries would be:

(1) compare winning \$10 (the initially established point) with losing \$10.

(2) compare winning \$20 with losing \$10.

(3) compare winning \$20 with losing \$20.

(4) compare winning \$40 or \$50 with losing \$20, etc.

This technique was used in Part I of the questionnaire; however, intuitively, it was reasoned that by representing the entries in such a progressive step procedure it would encourage the participant to establish the first probability and merely increase the probabilities by some predetermined increment. This would tend to defeat the purpose of having each entry determined solely on the basis of that entry alone. The order of the entries were made random to minimize this purely mathematical approach.

By application of Equation (8) through the logical progressive steps, all values of points from Part I can be calculated starting with the arbitrarily established point, which we have set as (+10,+1). Hereafter this procedure will be called the step-by-step procedure.

Referring again to the equation and Figure (1), it should be noted that each progressive step is based upon: (1) the calculated utility value for the previous step, and (2) the probability for the corresponding entry on the questionnaire. Thus, the validity of the points and the ultimate validity of the derived function equation depend upon the validity of each questionnaire entry. Part II was designed to provide a reference for comparing the probabilities and/or results of Part I. By allowing the participant to establish the monetary value of the win, there was no way of computing utiles step by step as was possible in Part I. Therefore, it was necessary to have a predetermined scale of values for reference. Part I furnished the only available scale source so the utility for each play on Part II was set equal to the utility reflected on Part I. Thus it can be seen that by our construction of the Part II questionnaire,

$$UX1(1) = - UN(1) \tag{21}$$

or:

$$UX1_j = - UN_j \tag{21a}$$

where $UX1_j$ is the utility value for the Part II point and UN_j is the value of the negative Part I point. Hereafter this procedure will be called the direct substitution method.

Figure (2) shows the calculated points of Part II as (0) points and calculated points of Part I as (Δ) points. Although Figure (2) reflects only the results of one questionnaire picked at random, the majority of the questionnaires

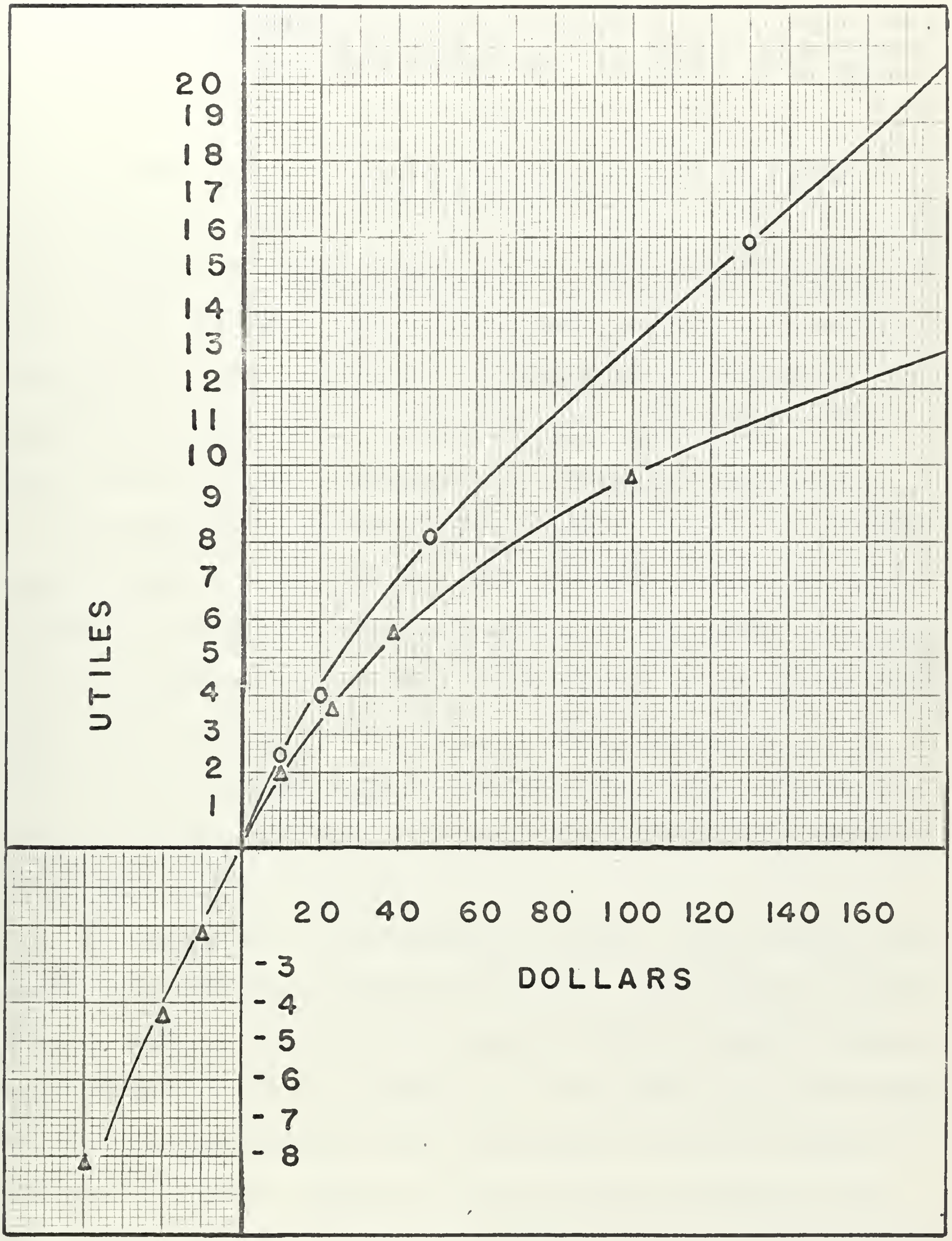

Figure 2

RELATIVE MAGNITUDES OF PARTS I AND II FOR GENERAL CASE

followed the example in that Part II gave a curve which had larger utility values for corresponding dollar values than did Part I.

At this point, several alternatives were available. We could use only Part I; we could use only Part II; or we could use some method of data smoothing to combine the results of Part I and Part II. Although conclusions were made on the comparison of the results of alternatives (1) and (2), it was deemed advantageous to utilize alternative (3) as the most logical method of analysis for reasons stated below.

It was noted earlier in this chapter that two distinct propositions were included in the questionnaire in order to provide a broader base for calculations. In order to retain the usefulness of having the two different types of questions, care must be taken to equalize, as much as possible, the weight given to each type in the final analysis. In the absence of full equality, at least the operator should know the weight given to each.

The equation for calculation of Part I data points gives complete weight to the Part I technique. The direct corrolation of Part II points to the negative Part I points probably incorporates 50% weight from Part I and 50% weight from Part II. If the data points were determined for all of Part I and then the data points for Part II were calculated from the results of Part I, any data smoothing process for these points would result in a higher weighting factor for Part I than Part II. To overcome this weighting factor completely

is impossible since it is necessary to use the values of Part I as a reference scale in determining Part II. Therefore any method which would introduce individual points of Part II, during a Part I type calculation would reduce the weighting factor of Part I.

A possible method of solution would be one which would calculate the dollar-utile points for Part I progressively from zero to the dollar point just immediately larger than the smallest dollar point for Part II. At this time, by means of a data smoothing technique, include the Part II point and recalculate all points on Part I to correct for this one point from Part II. Then the normal step-by-step method of calculation of Part I points would be resumed until the next larger Part II point was past. Recalculating all of the previous points before proceding would insure a more equalized weight from Part II.

Again several alternatives were available for projecting the points of the Part II curve into the final curve. An equation could have been determined for the Part I points; likewise, an equation could have been determined for the Part II points. Then a mean could have been found for all the points from these two equations. Several reasons rendered this solution impractical. If the equation included only the positive points of Part II, insufficient points were available to make this procedure practical. If the positive points of Part II and the negative points of Part I were used to determine the curve, the added weight factor for

Part I is still present. With this latter method, modification of the combined curve as well as the future Part II points after each separate point of Part II was incorporated into the combined curve would have been virtually unrealistic if done on a step-by-step basis. Several methods for calculating the final curve are explained below.

Part I Alternative

The simplest method would be to analyse just the Part I points. The dollar-utile points could quickly be determined by using the step-by-step procedure. The final utility curve could be determined by use of the various methods of regression analysis discussed in Chapter III. This alternative would be applicable for the participant who places more reliance on his responsiveness to the type of entries found on Part I or uses this type of reasoning in his daily business decisions.

Part II Alternative

As the name implies, this alternative utilizes the data from Part II. Part I points can be calculated by the step-by-step procedure. Then Part II points are calculated by the direct substitution procedure. To be useful, a curve must have both negative and positive points; therefore, the only effective way of developing this alternative would be to use the Part I negative points and the Part II positive points. This alternative is applicable for the participant who places more reliance on his responsiveness to the toss of a coin technique. It would also be applicable for the participant who normally makes business decisions of this type.

Part III Alternative

With this alternative, all of the points of Part II are incorporated with the Part I points. For the operator who does not have sufficient information to judge the participant's psychological reaction to the two different types of entries, this alternative would be most effective. Likewise in the case of a participant who places no added reliance on either method, this alternative would be most effective.

We tried two methods of data smoothing for this alternative. Both methods can be explained more easily by referring to Figure (3).

The first method is based upon the normal step-by-step calculation method for points of Part I with the periodic incorporation of the points for Part II. The normal step-by-step procedure for calculating Part I is started. After each point is calculated, the dollar value of that point is compared to the dollar value of the first Part II point. If the Part II point is larger, continue the normal procedure. When a Part I point is found to be greater than the first Part II point the normal procedure is terminated temporarily to incorporate the Part II point. Considering the incorporation of the first Part II point (UXl_1) on Figure (3), the function for Part II was supposed to be linear from point X_2 through point UXl_1 and projected to X_3'. The mean value of X_3 was calculated using the projected value X_3' and the normally calculated X_3. Because X_3' depends upon UXl_2, this latter value was recalculated. Now we are ready to continue

with the normal step-by-step procedure until the next Part II point is reached when the same recalculation procedure is made.

All previously calculated points should be recalculated whenever a Part II point is incorporated. Herein lies the difficulty of this method. To recalculate each previous point by this method would be very time consuming.

All the methods described in this chapter have used a step-by-step procedure for calculations. In the next chapter we shall describe in detail a method whereby the utilities of all questionnaire data points are computed simultaneously.

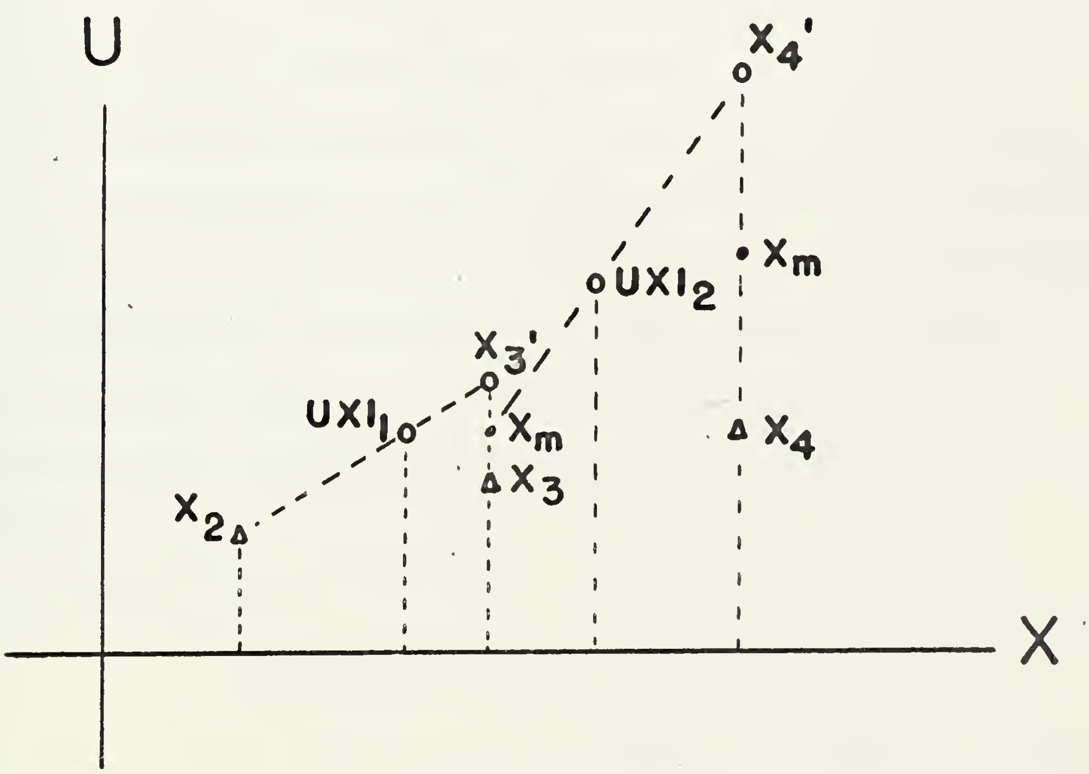

Figure 3

STEP-BY-STEP PROCEDURE SHOWN GRAPHICALLY

Chapter V

SIMULTANEOUS UTILITY CALCULATION

Because of the difficulties inherent with the step-by-step methods illustrated in the last chapter, we would like to develop a procedure which would make stepwise calculation unnecessary. We choose to do this by adopting an approach whose basis was that of the general least square concept.

In this method, we write a series of equations which will include each of the utility values (U_j, UN_j, $UX1_j$) whenever they appear at each data point. By solving these equations simultaneously, each value of the variables is considered in the calculations thus there is simultaneous calculation of each point in the curve-fitting process.

The preceding mathematical illustration and the discussion of the mathematical model have been based on the premise that the participant analyses each questionnaire entry with true indifference. Unfortunately, this premise is invalid. For this reason Equation (8) is not meaningful. We must incorporate the contribution of this non-indifference equation.

We start with:

$$U_{win} \; P_{win} - U_{lose} \; (1 - P_{win})$$

If we had a true indifference situation, the function would be set equal to zero and we would have Equation (8). However,

because we do not have true indifference, we set it equal to e_j, which represents the indifference error. Although the value of e_j is not known, it is hoped that it will be as close to zero as possible. The mathematical notation j denotes the jth equation in a series of equations, so we can write:

$$U_j P_j - U_j (1 - P_j) = e_j \quad . \qquad (8a)$$

where U_j is the utility value of the point and P_j is the probability of winning.

It was noted in the discussion of the step-by-step method that one of the utility values must be known in order to use Equation (8). From this it can be seen that the calculation process alternates between the use of a known positive value and the use of a known negative value. To make the calculations easier to follow, U_j has been designated as the utility of a positive point and PR_j as the probability of winning in the case where we proceed from a known positive U_j to determine the value of a negative utile. (These PR_j probabilities result from the non-equal money entries on Part I.) For the reverse procedure of calculating a positive U_j from a known negative value, UN_j has been designated as the utility value of the negative point and P_j as the probability when calculating an unknown positive value. (These P_j probabilities result from the equal money entries on Part I.) With the value for UN_j being negative, we can now write the equation for calculating a positive value as:

$$U_j \; P_j + UN_j \; (1 - P_j) = e_j \tag{8b}$$

and the equation when calculating a negative value as:

$$U_j \; PR_j + UN_j \; (1 - PR_j) = e_j \quad . \tag{8c}$$

In the discussion of the direct substitution method for calculating the Part II points, we suggested the equation:

$$UX1_j = - UN_j \tag{21a}$$

or:

$$UX1_j + UN_j = 0 \tag{21b}$$

where $UX1_j$ is the utility value for the jth Part II point and UN_j is the utility value for the jth negative point from Part I. Incorporating the indifference error, we get:

$$UX1_j + UN_j = e_j \tag{21c}$$

We have assigned a value to $U_1 = 1.0$ and called it the zeroth variable, so U_2 becomes the first variable or unknown. In order to establish the equations for the least square method, we will group together all of the equations of a type, for example, the equations used to determine a negative value from a known positive value. If we assume n number of data points for Part I and m number of data points for Part II, we can write our series of equations beginning with the first positive term, U_2, as follows:

$$e_1 = U_2 P_2 + UN_1 (1 - P_2) \qquad (B_1)$$

$$e_2 = U_3 P_3 + UN_2 (1 - P_3) \qquad (B_2)$$

$$\vdots$$

$$e_{n-1} = U_n P_n + UN_{n-1} (1 - P_n) \qquad (B_{n-1})$$

$$e_n = U_1 PR_1 + UN_1 (1 - PR_1) \qquad (B_n)$$

$$e_{n+1} = U_2 PR_2 + UN_2 (1 - PR_2) \qquad (B_{n+1})$$

$$\vdots$$

$$e_{2n-1} = U_n PR_n + UN_n (1 - PR_n) \qquad (B_{2n-1})$$

$$e_{2n} = UX1_1 + UN_1 \qquad (B_{2n})$$

$$\vdots$$

$$e_{2n+m} = UX1_m + UN_m \qquad (B_{2n+m})$$

The above set of equations does not represent the complete series for this method. The equation which will couple the UX1's to their neighboring U's will be included following a detailed explanation of the above equations. It is recognized that we have a sufficient number of equations above to solve for the unknowns by setting each equal to zero. However, use of e_j becomes a necessity when the coupling equations are added to this set.

Following the procedure described in the general mathematical model illustration, in the previous chapter,

the summation of the squares of the indifference errors:

$$E = \sum_{j=1}^{LL} (e_j)^2$$

where LL equals 2n + m - 1.

We now take the partial derivatives and determine at what points these derivatives are not equal to zero.

$$\text{(a)} \quad \frac{\partial E}{\partial u_k} = \sum_{j=1}^{LL} 2\, e_j \frac{\partial e}{\partial u_k} \qquad \text{from } k = 2,n$$

here $\frac{\partial e_j}{\partial u_k} = 0$, unless:

$$j = k - 1 \text{ where } \frac{\partial e_j}{\partial u_k} = P_k$$

or:

$$j = n + k - 1 \text{ where } \frac{\partial e_j}{\partial u_k} = PR_k$$

$$\text{(b)} \quad \frac{\partial E}{\partial UN_k} = \sum_{j=1}^{LL} 2\, e_j \frac{\partial e}{\partial UN_k} \qquad \text{from } k = 1,n$$

here $\frac{\partial E}{\partial UN_k} = 0$, unless:

$$j = k \text{ where } \frac{\partial e_j}{\partial UN_k} = (1 - P_{k+1})$$

or:

$$j = n + k - 1 \text{ where } \frac{\partial e_j}{\partial UN_k} = (1 - PN_k)$$

or:

$$j = 2n + k - 1 \text{ where } \frac{\partial e_j}{\partial UN_k} = 1.$$

$$(c)\ \frac{\partial E}{\partial UX1_k} = \sum_{j=1}^{LL} 2\, e_j \frac{\partial e_j}{\partial UX1_k} \quad \text{from } k = 1, nx1$$

here $\frac{\partial e_j}{\partial UX1_k} = 0$, unless:

$$j = 2n + k - 1 \text{ where } \frac{\partial e_j}{\partial UX1_k} = 1.$$

We have a total of LL number of equations in our illustration. The first (n-1) equations come from:

$$\sum_{j=1}^{LL} \frac{\partial e_j}{\partial U_k} e_j = 0.$$

The second (n) equations come from:

$$\sum_{j=1}^{LL} \frac{\partial e_j}{\partial UN_k} e_j = 0.$$

The last (m) equations come from:

$$\sum_{j=1}^{LL} \frac{\partial e_j}{\partial UX1_k} e_j = 0.$$

We now establish an array:

$$A_{1,1}Y_1 + A_{1,2}Y_2 + A_{1,3}Y_3 \ldots\ldots A_{1,LL}Y_{LL} = A_{1,LL+1}$$

where:

$$
\begin{aligned}
Y_1 &= U_2 \\
&\;\;\vdots \\
Y_k &= U_{k+1} && \text{where } k < n \\
&\;\;\vdots \\
Y_{n-1} &= U_n \\
Y_n &= UN_1 \\
Y_{n+1} &= UN_2 \\
&\;\;\vdots && \text{where } n \le k < 2n \\
Y_k &= UN_{k-n+1} \\
Y_{2n-1} &= UN_n \\
Y_{2n} &= UX1_1 \\
Y_{2n+1} &= UX1_2 \\
&\;\;\vdots && \text{where } 2n \le k < 2n+m \\
Y_{LL} &= UX1_m
\end{aligned}
$$

(U_2 is the first unknown because $U_1 = 1.0$ is known.)

To exemplify this procedure, we now look at the array positions which receive values from the partial derivative $\frac{\partial e_j}{\partial U_3}$.

$$\sum_{j=1}^{LL} \frac{\partial e_j}{\partial U_3} e_j = 0$$

but this is equal to:

$$\frac{\partial e_2}{\partial U_3} e_2 + \frac{\partial e_{n+2}}{\partial U_3} e_{n+e} = 0$$

(All other derivatives in the sum are zero.) Note that the second equation comes from the derivative with respect to U_3 because U_2 is the first unknown, U_3 the second, and so forth.

As $$e_2 = U_3 P_3 + UN_2 (1 - P_3)$$

and $$e_{n+2} = U_3 PR_3 + UN_3 (1 - PR_3)$$

the second equation is:

$$(PR_3^2 + P_3^2) U_3 + P_3 (1 - P_3) UN_2 + PR_3 (1 - PR_3) UN_1 = 0.$$

so
$$\begin{aligned} A(2,1) &= 0 \\ A(2,2) &= PR_3^2 + P_3^2 \\ &\cdot \\ &\cdot \\ A(2,n) &= PR_3 (1 - PR_3) \\ A(2,n+1) &= P_3 (1 - P_3), \text{ etc.} \end{aligned}$$

The next portion of the discussion will involve the procedure to be used to incorporate the non-indifference equations. It will be noted that there is no way to predict where the Part II points will lie in relation to the X_j points; therefore, the same index limits which were used in the (b) equations do not apply here. We will use a sub-program to search for the relative locations of the Part II points. After establishing the location of each Part II point as on or between two identified Part I points, we can

[illegible]

[illegible]

[illegible]

$$[illegible]$$

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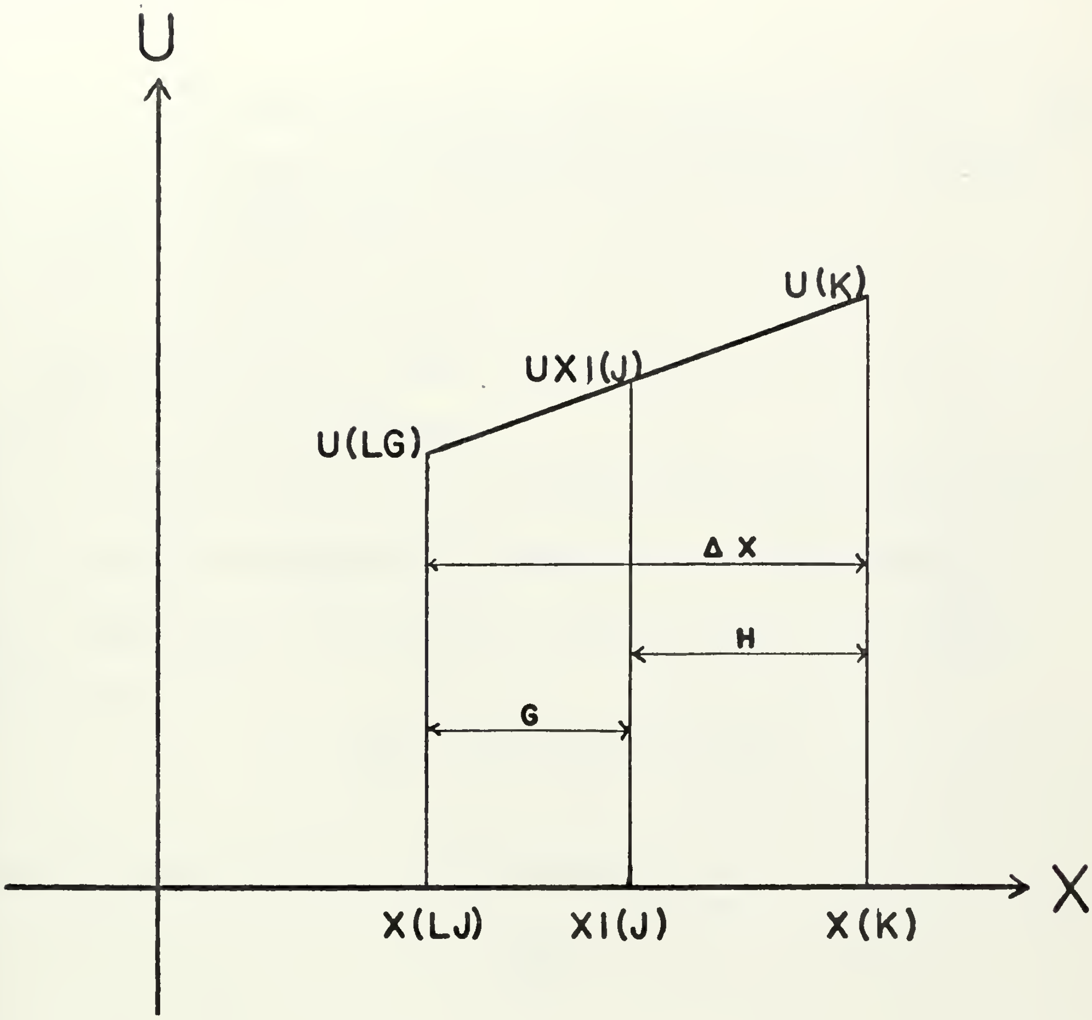

Figure 4

GRAPHIC MODEL FOR THE COUPLING EQUATION

write an equation which will enable us to add the contribution of this Part II point to the immediately associated Part I points. In Figure (3), a point from Part II ($X1_j$) lies between two points from Part I (K_k and X_{k-1}). We will consider the function is linear between these three points. Then that e which represents the added contribution from the Part II point is given by:

$$e_j = U_k \left[\frac{\Delta g}{\Delta X}\right] + U_{LJ} \left[\frac{\Delta h}{\Delta X}\right] - UX1_j \tag{22}$$

where LJ - k - 1.

The corollation between the (b) equations and the coupling equation can best be understood by reference to Figure (3). $X1_j$ will affect the:

(LJ - 1)th equation, the

(LJ)th equation and the

(2n + j - 1)th equation,

where the value of j has no connection with the value of LJ. The only known relationship is that $X1_j$ is immediately greater than $X(_{LJ})$. The contribution to the (LJ - 1)th equation is:

$$e_j \left(\frac{h}{\Delta X}\right) \quad \text{where } \left(\frac{h}{\Delta X} = \frac{\partial e_j}{\partial U_{LJ}}\right)$$

The contribution to the (LJ)th equation is:

$$e_j \left(\frac{g}{\Delta X}\right)$$

and the (2N + j - 1)th is:

$$-e_j.$$

For illustrative purposes we shall apply the above equation to the case where the 2nd X1 (j=2) point lies between X_3 and X_4 (thus LJ will equal 3) and n equals 5. The contribution of this e to the (LJ - 1)th equation will be:

$$\frac{h}{\Delta X} \left[U_{LJ+1} \left(\frac{g}{\Delta X}\right) + U_{LJ} \left(\frac{h}{\Delta X}\right) - UX1_j\right]$$

and to the (LJ)th equation:

$$\frac{g}{\Delta X} [U_{LJ} (\frac{g}{\Delta X}) + U_{LJ-1} (\frac{h}{\Delta X}) - UX1_j]$$

and to the (2N + J - 1)th equation:

$$-[U_{NNJ} (\frac{g}{\Delta X}) + U_{NNJ-1} (\frac{h}{\Delta X}) - UX1_j]$$

where (2n + j - 1) = NNJ.

These separate expressions must be added to the term already computed for the three equations. In order to better show the effect of this contribution upon some of the same array positions as used by the previous example, we will set LJ equal to 3. The following would be added to the (LJ-1)th equation in the array positions as indicated:

$$A(_{2,1}) = 0$$
$$A(_{2,2}) = (h/\Delta X)^2$$
$$A(_{2,3}) = (h/\Delta X)(g/\Delta X)$$
$$A(_{2,4}) = 0$$

.

.

$$A(_{2,1}) = -(h/\Delta X), \text{ etc.}$$

and to the (LJ)th equation:

$$A(_{3,1}) = 0$$
$$A(_{3,2}) = -(h/\Delta X)(g/\Delta X)$$
$$A(_{3,3}) = (g/\Delta X)^2$$
$$A(_{3,4}) = 0$$

.

.

$$A(_{3,11}) = -(g/\Delta X)$$

and to the (2n + j-1)st equation:

$$A(_{11,1}) = 0$$

$$A(_{11,2}) = (h/\Delta X)(g/\Delta X)$$

$$A(_{11,3}) = -(g/\Delta X)$$

$$A(_{11,4}) = 0$$

$$\cdot$$

$$\cdot$$

$$A(_{11,11}) = 1.$$

We have only shown the contribution from the non-indifference equation. The total value for any of the array positions will be the array entry previously calculated plus the contribution.

All other Part II points for which $X_n > Xl_j$ are incorporated in a similar manner. Once these values are determined a library routine is used to solve the simultaneous equations and compute the utile values for each dollar point.

Thus the calculation of the utility curve for an individual can be accomplished by whichever method the operator believes will give the more accurate results for that individual. Further experimentation may generate sufficient data which could lead to some qualitative conclusions on the use of the several alternatives.

Chapter VI

RESULTS OF QUESTIONNAIRE RESEARCH

The Questionnaire

Members of the staff and fellow students in the department were asked to complete the questionnaire in the initial development period. When the final revised version was adopted, about 200 questionnaires were distributed to selected groups in the School of Business, University of Kansas. Those questionnaires, which were completed and returned by these volunteer students, form the basis for this research.

The purpose of distributing the questionnaire was not to obtain statistical data on the students. We were interested in testing the usefulness of the questionnaire as a practical means of obtaining data. Our second purpose was to investigate whether this data would be suitable for computer analysis.

The scope of this chapter has been limited to the questionnaire, itself, and not to the volunteer students. We analysed the questionnaires to see if we could identify those types of errors which could be corrected in the future. Some of the more important types of errors have been documented in this chapter. We shall finish the chapter with a brief resumé of the success obtained in adapting the data to various methods of curve-fitting.

In order to have acceptable data for the computer programs, a set of minimum standards or limitations were established. Those responses or questionnaire entries which

do not meet the minimum standards will be identified by type of error in the subsequent analysis of the data.

It should be noted here that the purpose for the questionnaire was to create a workable aid for business executives to introduce the element of risk into business decisions. We shall make frequent reference to this purpose when we discuss the types of errors found on the research questionnaires.

Any questionnaire with less than 4 equal money bets and 4 non-equal money bets was not analyzed. This number of points was considered the absolute minimum required to give any realistic results. This type of error was more prevalent than any other type. Two participants made the comment that based upon their present financial status they could not afford to make bets over $10 or $20. Others considered the monetary figures unrealistic for a student group. This criticism has value, however, it is believed that this reasoning reflects, in general, a misunderstanding of the value of money. Four bets would involve the maximum loss of $100. Although the participants did not have this amount of ready cash to bet, it is believed that any of them would have been tempted to borrow this sum in case of loss for the expected value of return when the odds were 5 or 10 to 1. In the cases where the participants are business executives, this type of error probably will not exist.

It was surprising to find so large a percentage of questionnaires which reflected mere carelessness or a general misunderstanding of the Part I type of question. Several

cases contained probabilities of winning of 25% in over one-half of the entries; these probabilities were noted in the lower monetary bets. One of the students indicated in his comments that he wondered if we were trying to quiz him on his knowledge of the course in probability. This reaction was reflected in another case where a probability was shown as 28 4/7.

Another type of error in this general area was the isolated entry which was completely unrealistic when considered with the other entries. Some of these cases probably can be attributed to mere carelessness where the participant entered the probability of losing vice winning. For this reason alone, it is believed that at least half of the entries in this category could have been corrected if the questionnaire had been reviewed with the participant when he finished.

Another problem area resulted when the probabilities for $10 values were between 85% and 90%. The resultant curve in such a case is unrealistic and would serve no practical purpose because it becomes virtually asymptotic to a dollar value of very small magnitude. Although this might in fact reflect human behavior for a very conservative person, probabilities of this magnitude appear unrealistic for business students and most likely would not exist in the case of business executives.

A minimum number of participants simply would not make an even bet, while one person would make only even bets. In each case, the entries on Part II were consistent with the

answers reflected on Part I. The present mathematical solution for the data from the questionnaire will not work in either of these cases.

Although the proportion of questionnairs which had to be rejected was higher than anticipated, the response, in general, to the questionnaire was satisfactory. The percentage of cases resulting from apparent misunderstanding might well reflect on the instructions to the questionnaire. In the revision period, the instructions had been redrafted several times to incorporate the constructive criticisms of the early subjects. Even with the indicated response of the students, we believe that the instructions contain sufficient, easily understood details. Likewise we believe the instructions to be of optimum length in that they contain sufficient detail and yet are not too long to cause an initial adverse reaction from the participant.

The Curve-Fitting Processes

The next step in the analysis concerned the curve-fitting process on the data from the questionnaires. The use of only Part I points proved more successful in the calculation of the two constants of our general mathematical equation. This had been expected. However, the results even in this area were dependent to a large degree upon the nature of the curve. Those questionnaires which contained probabilities close to the even bet situation were the easiest to fit. For those cases where the later bets were in the 90's, the slope of

the curve changed radically and rapidly upward. Unfortunately, each of the programs attempted to fit the end points of the curve and left the greater deviation in the area where we would be concerned the most. By disregarding these latter points, the curve-fitting results were much better. This procedure is not without merit. The sudden increase in probability could indicate that the monetary gain or loss probably has exceeded the point at which the person has a realistic appreciation for the value of that monetary amount.

The results of the combined curve of Part II points with Part I negative points depended generally upon the number of points on Part II. As the number of points increased in quadrant III, the general tendency was for the amounts to increase very rapidly around the 7th to 8th point ($1000 to $2000). Were the number of points on Part II to equal this number, the sharp incline of the curve resulted as it did in Part I. Generally the utility obtained for Part II points was greater than the utility of the corresponding Part I points. This factor apparently made the curve more nearly the normal logarithmic curve because it was noted that over the same range of dollar values the standard deviation was generally less for Part II points than with use of Part I points.

The combination of Part II points with Part I points by the step-by-step method created a series of steps in both the first and third quadrants. Although the calculated curve, using Equation (4), probably was a relatively smooth

curve, the standard deviation was much higher than the case when only Part I points were used. This step departure from the normal curve had its adverse effect on the calculation of an equation in each of the programs. The continual change of the slope made curve-fitting difficult.

Of all the programs, the FOFB approach responded better than the others. SNOLSE was dependent, to a large degree, upon the accuracy of the initial guesses for the coefficients in the case where the curve was erratic. If the deviation of the guesses were greater than about 15%, the program tended rapidly to increase the value of the coefficients until the size of the coefficients exceeded the limitation of $1 \times 10^{\pm 15}$. The original logarithmic program was the least responsive of all the programs. With calculated data, the program would give extremely accurate results if the initial guess for BTRY was close to the true B. However, even with calculated data when the initial guess for BTRY had a 20 to 25% deviation from the actual B, the coefficient B was made progressively larger as was the case of SNOLSE.

Our results were indecisive as to the number of coefficients to be used. We tried to use the equation

$$U_j = a_1 \ln ((x_j + B)/B) + a_2 x_j + a_3 (x_j)^2$$

with the SNOLSE program. A majority of cases gave the best fit with the use of only the first term containing the constants a_1 and b. In a few isolated cases the inclusion of the a_2 term gave better results. The usual result with

the use of the a_2 terms was that the a_1 coefficient would increase about two fold before a singular matrix position was indicated. With the use of the a_3 term, two results were common. The coefficient a_3 became so large or so small that its value would exceed the limitation of $1 \times 10^{\pm 15}$ and give us no answer. The second result was a singular matrix.

In the cases where the curve made rapid ascent in the first quadrant, a straight least square program generally proved unacceptable. Better results were obtained with the use of our basic equation in these cases than with the use of a straight least square. The standard deviation was larger when we used the basic equation in these cases, but with the use of the least square method, we found several instances where the standard deviation increased with an increase of coefficients. This indicated some error in the application of least square to this problem case.

To insure no misunderstanding on the use of terms, our discussion in the previous paragraph was on the general least square method of calculating coefficients. Hereafter, we shall analyze the effectiveness of the special method which we identified as the least square method of incorporating the two sets of data points. As a generality, it can be concluded that our least square method was more satisfactory than the step-by-step method. Of foremost importance is the fact that the calculated data points created a smoother curve. This factor made curve-fitting easier. The simultaneous calculation procedure gives equal weight to each

value of the variables and thus offers the maximum equality to both Part I and Part II points.

Some difficulties and some unresolved questions were encountered with our least square method. In about one-fourth of the cases, the method was unstable. The instability was indicated by a singular matrix as an answer or a change of sign for all of the utile points. It is believed that the cause for this difficulty arises from the inability of the general least square approach to respond satisfactorily to erratic data points. It was noted that in some cases the calculated utile values for corresponding dollar points were from two to three times larger with the least square method than with the step-by-step method. However, this factor proved to be of value in that the resulting curve was more realistic than the curve derived by the other method.

Chapter VII

SUMMARY AND CONCLUSIONS

The research conducted here has been for the purpose of investigating the application of utility theory as an aid for making business decisions. This investigation seemed valuable because other methods fail to incorporate the element of risk in a reasonable way. As it evolved, the work focused on ways to make the utility calculation a mechanical process.

In order to mechanize the procedure two things were required. One, a curve-fitting method which could be expected to work for all reasonable utility curves. Two, a way of producing data for the use of the curve-fitting process. These two combined would produce a mathematical expression for any utility curve.

After such a mathematical function is available, an investigation of the usefulness of the utility method would be possible. Because of time limitations, this work concerned itself only with the two preliminary requirements.

A questionnaire was designed which would generate probabilities from a series of questions involving two different games of chance. These two games were represented separately as Part I entries and Part II entries. The final draft of this questionnaire was distributed to selected groups of business students at the University of Kansas. The purpose of the distribution was limited to the analysis of the effectiveness of the questionnaire as a means of generating useful data which could be converted to utility values.

In our case with students as subjects, the effectiveness of the questionnaire proved disappointing. However, those problem areas of misunderstanding by the students should be reduced considerably when the questionnaire is administered to business executives. With the latter subjects, the monetary values of the entries should not exceed the monetary values of the business ventures which these executives consider on a routine basis. Likewise the interpretation of probability as applied to alternative business ventures should be understood with more ease and clarity. It was apparent, however, that the questionnaire is more effective when administered in the presence of a qualified operator. It can be concluded that when the problem areas of misunderstanding are reduced to a minimum, the questionnaire will be an effective means of generating data.

In the development of the questionnaire it was recognized that individuals respond differently to different games of chance or types of bets. For this reason, two separate types of propositions were included. An effective means of incorporating the data from these different propositions created one of the major problems in this research work.

As noted earlier, the use of only Part I data proved to be the easiest. Use of Part I negative data and Part II positive data likewise created very little difficulty. We found that participants generally placed a higher utility for corresponding dollar entries with the Part II proposition than with Part I. Because of this factor, the combined use of Part I and Part II data proved more difficult.

Two methods are presented for the incorporation of the Part II points with the Part I points. The first method, identified as the step-by-step procedure, was responsive for all questionnaires which were included in this portion of the analysis. Unfortunately, this method did not give a smooth curve. A modification of the method would be to recalculate all previous points whenever a Part II point was incorporated. To recalculate each previous point by the step-by-step procedure, however, would be very time consuming. Herein lies the difficulty of this method.

Because of the difficulties inherent with the step-by-step methods, we developed a procedure which would make stepwise calculations unnecessary. We chose to do this by adopting an approach whose basis was that of the general least square concept. A series of equations was written which included each of the utility values whenever they appeared at each data point. By solving these equations simultaneously, each value of the variable was considered in each of the calculations. As a result there was simultaneous calculation of each point in the curve-fitting procedure. Although this method is considerably more accurate and superior to the step-by-step procedure it was not responsive in all cases. It is believed that the inability of the general least square approach to respond satisfactorily to erratic data points caused the difficulties of this method. In those cases where the data points represented a relatively smooth curve, this method was responsive and proved most satisfactory.

It is concluded that our least square method is the more accurate and should be used in all cases where the program is responsive. The step-by-step procedure might be used in the other cases.

The actual curve-fitting portion of the research included two different procedures. The general least square method was used to attempt to fit the curve with a logarithm function. This procedure was found to have limited stability. It is believed that the lack of stability may be attributed to the same reasons that we noted earlier concerning the least square approach.

An iterative procedure was used to find two values for the unknown variable so that the first derivative of one was greater than zero while the other was less than zero. This method, identified as the twenty question method, proved more useful. One possible source of difficulty is shown when the largest negative dollar value is quite small. In these cases, the iterative process of finding a value which will make the first derivative less than zero is quite lengthy. Minor modifications to the program should reduce this difficulty and can be made for those specific cases.

The results of the research of the data which was generated by the questionnaire have not been as conclusive as had been hoped when the project was commenced. Although the questionnaire approach has avoided any direct mention of human behavior, this element has been thoroughly apparent in the probabilities. The inclusion of this factor, human

behavior, in the probabilities is shown both in the erratic progression of the data points and in the Part I and Part II first quadrant points. Each of these results has increased the difficulties of curve-fitting and obtaining consistent results. Because of these difficulties, conclusive statements are not justified, without further research, as to the type of curve or type of program should be the most effective with the questionnaire.

Although this work has concerned itself only with the development of the mathematical expression for a utility curve, the limited analysis of the data justifies the following comments. The several computer programs can be used to calculate the utility function; then the best curve can be used to plot the function. As an alternative, the actual utile-dollar points can be readily plotted by a library computer program or easily done manually. This plotted utility curve has several fold advantages for the decision maker. For example, a method of introducing risk as a substitute for pure hunch and intuitive reasoning to the decision is available with the use of the utility curve. The questionnaire-computer package offers a quick means for revising the criteria for decision making to keep step with the rapidly changing economic status of the company or of the community. It offers a ready reference for subordinates to make decisions for management which would be consistent with general management policies. Likewise, management, itself, has a ready reference which could be used to add consistency to the decision.

This work may form the basis for further research which could develop methods of using the utility theory to make simple business decisions by computers. For further research in the investigation of the usefulness of the utility method, these recommendations are made. More extensive tests with the suggested least square method of data smoothing of Part I points and Part II points are recommended. Likewise extended research into the adaptability of investment type questions into the questionnaire is recommended.

The questionnaire-computer package has not been perfected to the extent that it is the ready answer for business executives faced with decision making. However, the uses of this package as a workable aid to decision making should prove of assistance in delegating authority to subordinates and in obtaining consistency in decisions rendered.

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APPENDIXES

APPENDIX I

THE QUESTIONNAIRE

Appendix I

The purpose of the thesis is to investigate the posibilities of the use of the utility theory in decision making. For this reason we would like to stress that the date must represent the bets you would be willing to make based upon your personal financial position TODAY. Although we will not attempt to solicit an actual bet from you based upon your answer, this might well be the best criterion upon which to base your answers.

Since the first draft, several personal factors of the volunteer have been deemed important to the final analysis of the data. It is, therefore, requested that you answer the additional questions listed below. This information will be considered confidential; any data that might be published would be by groups, such as married juniors, single seniors, etc.

Year group in school:
- Junior ____________
- Senior ____________
- MBA 1st year ______
- MBA 2nd year ______
- Other _____________

Marital status:
- Single ______
- Married _____

Number of Dependents __________

Male __________
Female ________

Please place an X beside the type of housing you occupy at the University:
- Dormitory ____________________
- Fraternity or Sorority ___
- Live in property which you own __
- Live in rental property __
- Live at home with parents _______

Financial Status: (probably the most important question on the list)
- Approximate yearly income (include spouse's income) ______
- Are you paying all of your own school expenses? __________
- Are you on a scholarship? _____ If so, what percent of your school expenses does this represent? _____
- Are you receiving other financial aid while in school? ___ If so, what percent of your school expenses does this represent? _____
- Are you working part time while attending school? _____

The author welcomes any constructive comments or criticism on this questionnaire. Such comments can be made on the back of this sheet.

UTILITY CURVE QUESTIONNAIRE

This questionnaire is part of an investigation of the utility theory as applied to decision making. It consists of a series of questions about investments. To simplify the situation, however, a game of chance will be substituted for investments. In the questions you will be asked to estimate the percent probability, that your opponent will not draw a winning card, which you would require in order to ACTUALLY make the series of bets. It is hoped that these probabilities can be used to evaluate an investment of equal monetary value.

To insure the greatest validity of the questionnaire, it is requested that you base your answers on these two criteria. First, your answer should reflect the EXACT probability which you would require to make the bet. Secondly, in the spirit of the competitiveness of the business world, your answer should reflect the MINIMUM probability you would require. This latter criterion is associated with the practice of the lowest bidder being accepted in a call for sealed bids.

For Part I of the questionnaire, the game will be played with a deck of 100 cards consisting of black cards and red cards. The results of the game depend upon one and only one card being drawn by your opponent. Should your opponent draw a BLACK card, you WIN; should he draw a RED card, you LOSE. You can designate the number of black cards in the total deck of 100 to establish the conditions under which you would be willing to bet that your opponent will not draw a red card and win the bet. It might prove helpful in those cases where you are in doubt as to a satisfactory answer to follow the below listed series of questions, or similar questions, until an acceptable ratio is found.

Would you bet that your opponent would not:

(1) draw a black card out of an ordinary deck of playing cards? If so, the probability in our game would be 50.

(2) draw a spade? If so, the probability would be 75.

(3) draw an ace? If so, the probability would be 92.

(4) draw the Ace of Spades? If so, the probability would be 98.

In some cases you may not want to make the bet even with 99% probability that your opponent would not draw a winning card. In those cases, please indicate NO BET as your answer.

PART I OF QUESTIONNAIRE

Amount of Win	Amount of Loss	Percent Probability
$ 10.	$ 10.	________
200.	100.	________
1,000.	1,000.	________
20.	10.	________
2,000.	2,000.	________
20,000.	10,000.	________
20.	20.	________
100.	40.	________
4,000.	4,000.	________
40.	20.	________
20,000.	20,000.	________
40.	40.	________
4,000.	2,000.	________
10,000.	10,000.	________
400.	200.	________
100.	100.	________
2,000.	1,000.	________
200.	200.	________
1,000.	400.	________
400.	400.	________
10,000.	4,000.	________

PART II OF QUESTIONNAIRE

On this second part of the questionnaire, the game consists of a series of bets on one toss of a fair coin, so you will have a 50/50 probability of winning. You are asked to indicate in the second column the amount you would expect to win in order to play the game on the basis of the amount of loss indicated in column (1). You are betting on an even probability that your opponent will not win the toss on one and only one toss of the coin. Remember, you have an even chance of winning or losing. Your answer may be recorded as 100 to indicate a $100 bet.

Amount of Loss	Amount of Win	Percent Probability
$ 10.	______	50/100
4,000.	______	50/100
20.	______	50/100
1,000.	______	50/100
20,000.	______	50/100
100.	______	50/100
2,000.	______	50/100
40.	______	50/100
200.	______	50/100
10,000.	______	50/100
400.	______	50/100

APPENDIX II

FORTRAN PROGRAM FOR LEAST SQUARE METHOD

Appendix II

COMPUTER PROGRAM TO CALCULATE UTILITY VALUES BY THE LEAST SQUARE METHOD

Purpose

The purpose of this program is to incorporate the Part II data points with the Part I data points. The data smoothing technique used is that of solving a series of equations simultaneously. The utile-dollar data points derived by this program can be used as input data for one of the regression analysis programs.

Language

Fortran IV (IBM 7040 Computer).

Symbolic Dictionary

VARIABLE	S/A*	I/O**	DESCRIPTION
N	S	I	Total number of Part I data points.
NX1	S	I	Total number of Part II data points.
PR	A	I&O	Probabilities used to determine a positive utility value from a negative utility value.
PRN	A	I&O	Probabilities used to determine a negative utility value from a known positive utility value.
X1	A	I	Dollar values for the Part II data points.
X	A	I	Dollar values from questionnaire entries which become the dollar values for the positive Part I points.

*S - Single variable; A - Array of variables.

**I - Input; O - Output.

U	A	0	Utility values for the positive Part I data points.
UN	A	0	Utility values for the negative Part I data points.
UNX1	A	-	Utility value for the Part II data points.

Program Routine

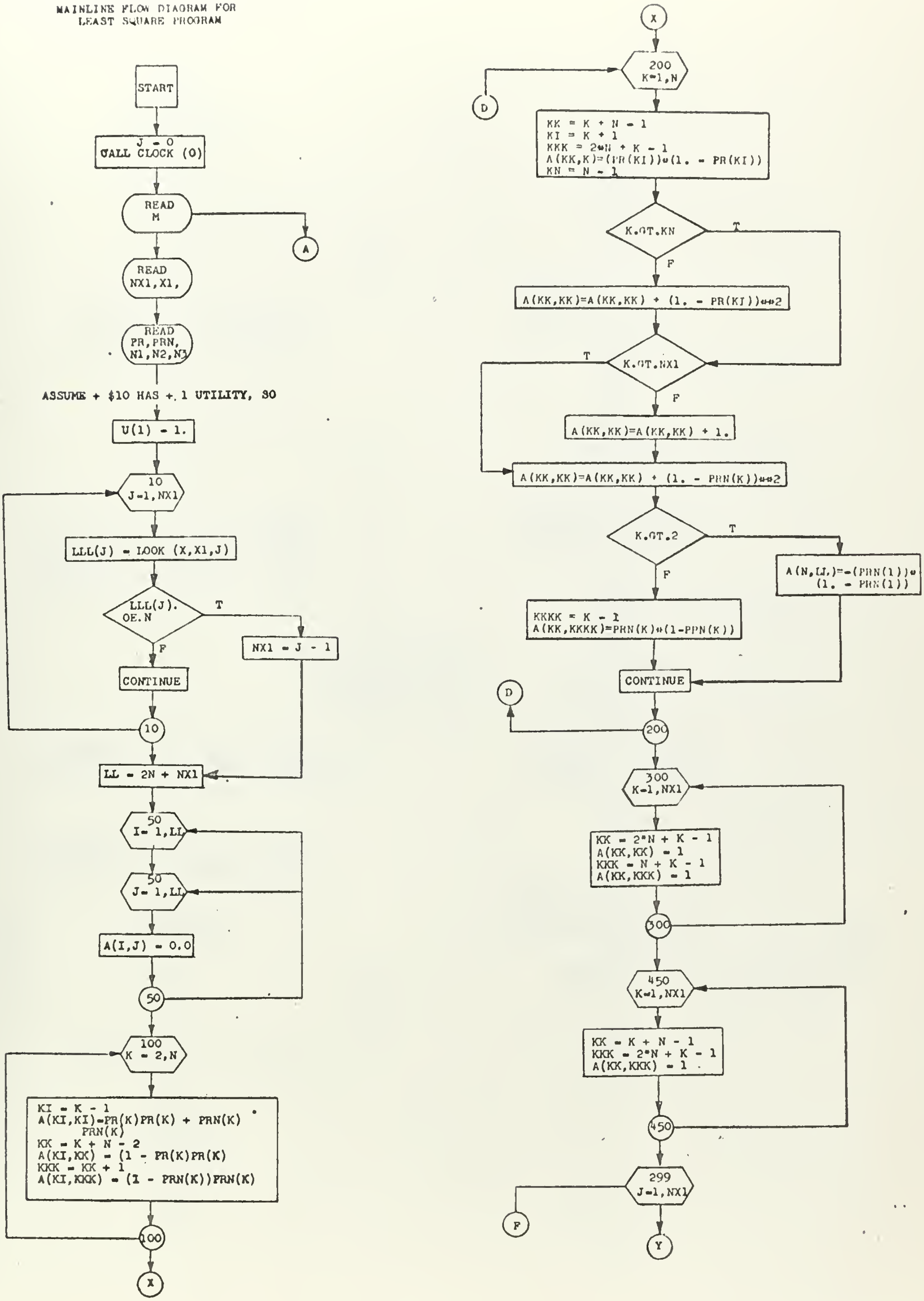
This program uses the general least square concept to incorporate data points from two sources. It is divided into two sections, namely, the matrix generater and the matrix solver.

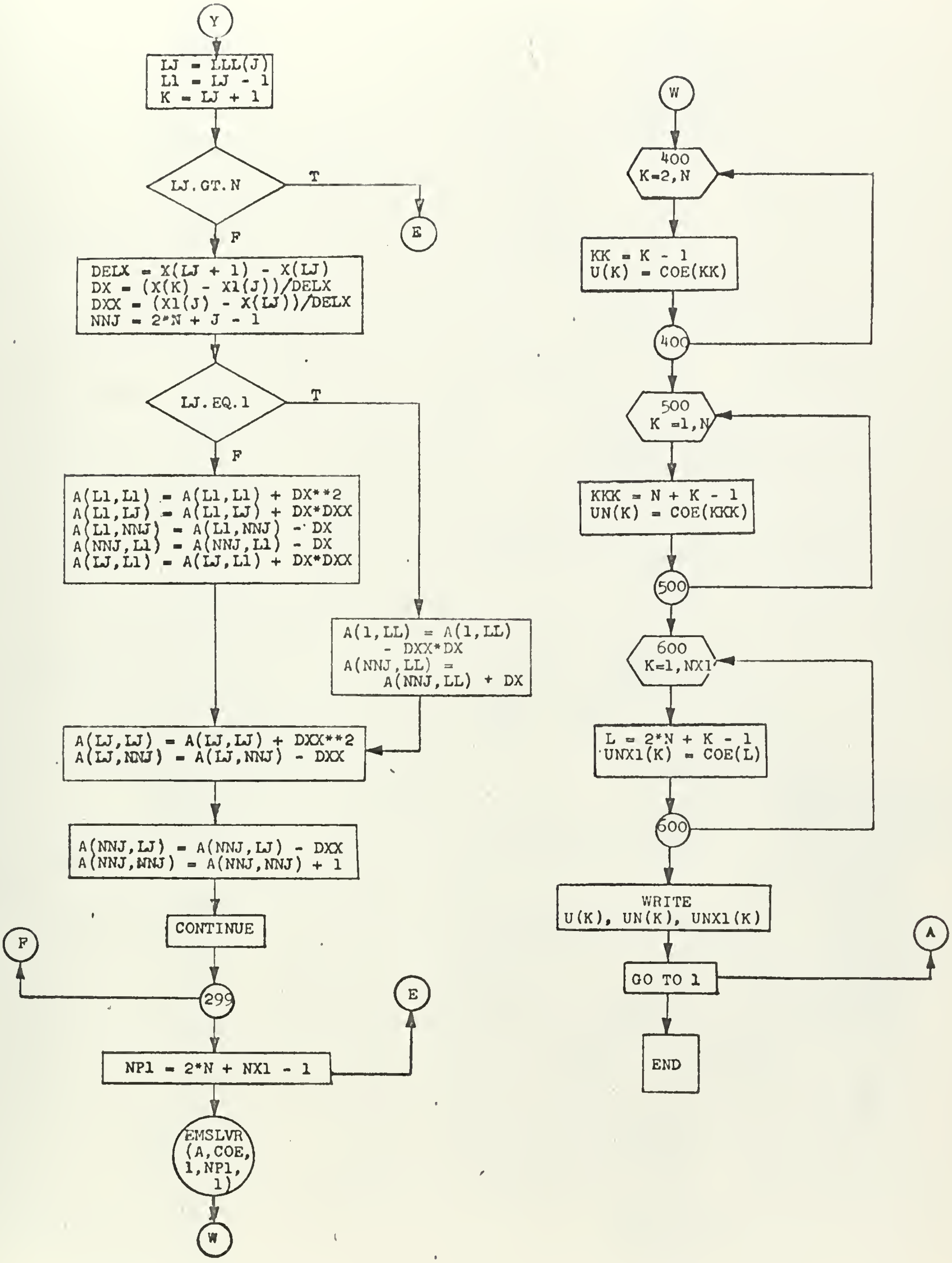
To develop the matrix generater, the following steps were used. A series of equations were written which invluded each of the utility values (U_j, UN_j, $UNX1_j$) whenever they appear at each data point. The partial derivatives of each equation with respect to the three variables were established. A square array with dimensions equal to the number of equations was used to represent the matrix generater. The rows within the array represented the derivatives of the indifference equations starting with e_1 through e_j. The columns represented the various variables and were designated from left to right as U_2 through U_n, UN_1 through UN_n, $UNX1_1$ through $UNX1_m$, and the last column contained the known value U_1. (For this explanation only, n equals the number of Part I data points while m equals the number of Part II data points.) Whenever a partial derivative was not equal to zero, its coefficients were computed into the proper array position. Once all of these values were entered, a library program was used to solve the matrix.

By solving these equations simultaneously, each value of the variables is considered in the calculation thus there is simultaneous calculation of each point in the curve-fitting process.

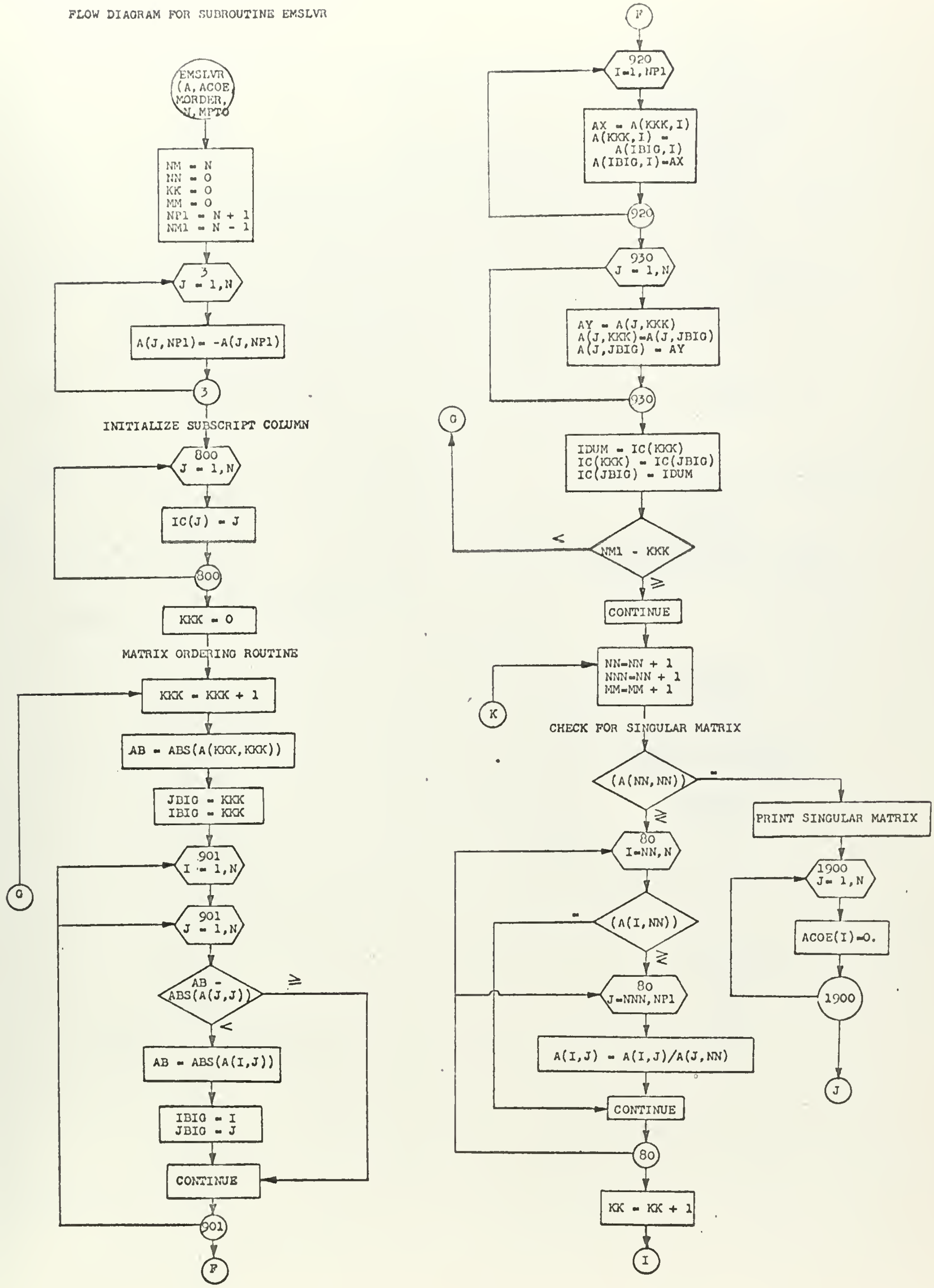
As this program is written, all dollar values must be transferred to one array and ordered from largest negative to largest positive before this data can be read into either of the regression analysis programs. The corresponding utility values are ordered accordingly.

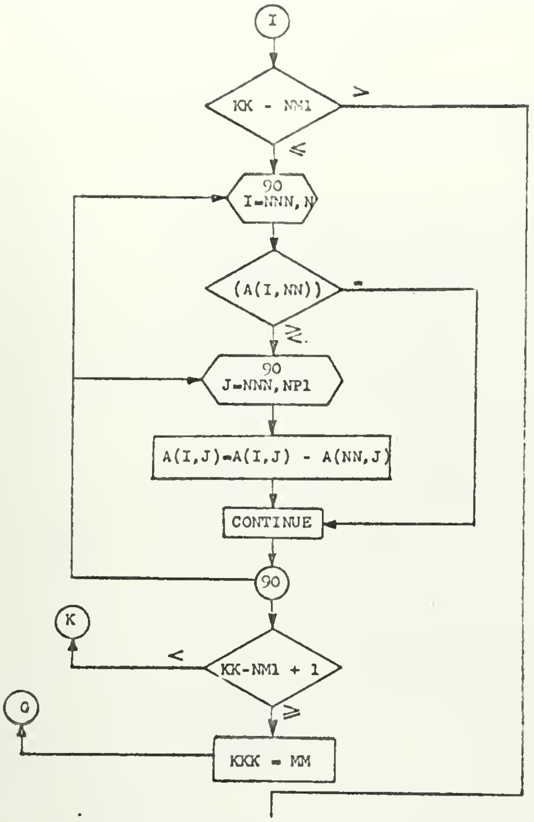
MAINLINE FLOW DIAGRAM FOR
LEAST SQUARE PROGRAM



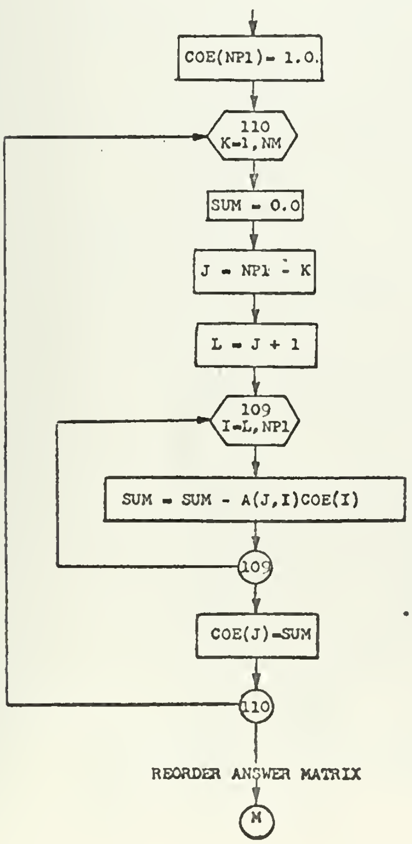

Y
LJ = LLL(J)
L1 = LJ - 1
K = LJ + 1
LJ.GT.N
T
E
F
DELX = X(LJ + 1) - X(LJ)
DX = (X(K) - X1(J))/DELX
DXX = (X1(J) - X(LJ))/DELX
NNJ = 2*N + J - 1
LJ.EQ.1
T
F
A(L1,L1) = A(L1,L1) + DX**2
A(L1,LJ) = A(L1,LJ) + DX*DXX
A(L1,NNJ) = A(L1,NNJ) - DX
A(NNJ,L1) = A(NNJ,L1) - DX
A(LJ,L1) = A(LJ,L1) + DX*DXX
A(1,LL) = A(1,LL) - DXX*DX
A(NNJ,LL) = A(NNJ,LL) + DX
A(LJ,LJ) = A(LJ,LJ) + DXX**2
A(LJ,NNJ) = A(LJ,NNJ) - DXX
A(NNJ,LJ) = A(NNJ,LJ) - DXX
A(NNJ,NNJ) = A(NNJ,NNJ) + 1
CONTINUE
F
299
E
NP1 = 2*N + NX1 - 1
EMSLVR (A,COE, 1,NP1, 1)
W
W
400 K=2,N
KK = K - 1
U(K) = COE(KK)
400
500 K =1,N
KKK = N + K - 1
UN(K) = COE(KKK)
500
600 K=1,NX1
L = 2*N + K - 1
UNX1(K) = COE(L)
600
WRITE U(K), UN(K), UNX1(K)
A
GO TO 1
END

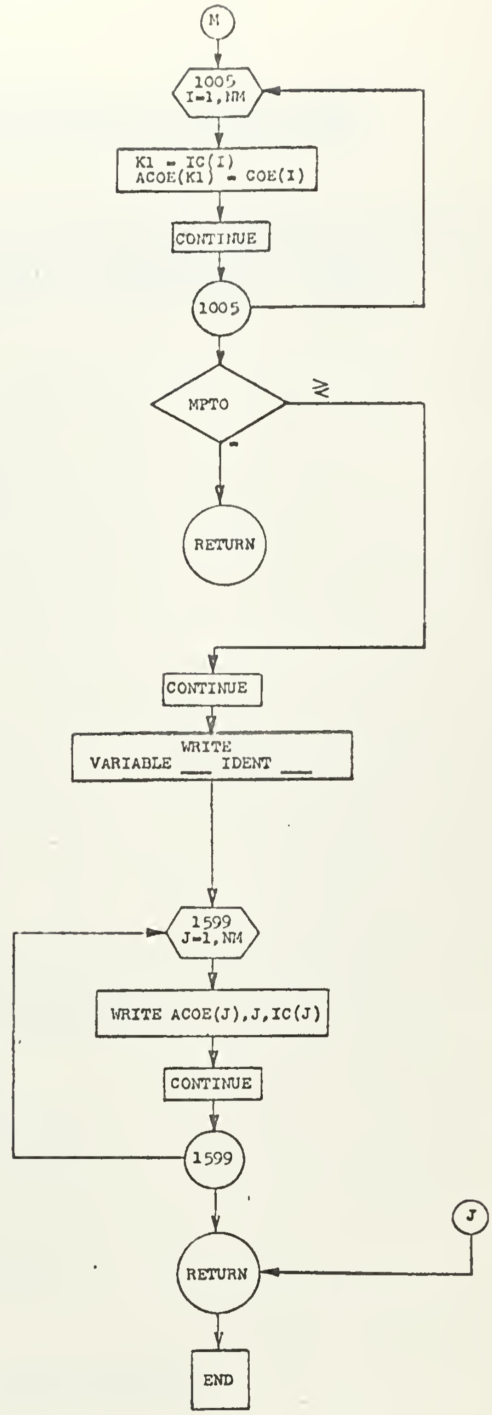
FLOW DIAGRAM FOR SUBROUTINE EMSLVR





BACK SOLVE UPPER TRIANGULAR MATRIX





```
C     THE LEAST SQUARE PROGRAM DESIGNED TO INCORPORATE
C     PART II POINTS BY USE OF SIMULTANEOUS EQUATIONS.

      DOUBLE PRECISION A,PR,PRN,DELX,X,DX,DXX,U,COE,UN,UNX1,X1,P
      DIMENSION UNX1(50) , COE(50),LLL(50),X1(50),X(50)
      DIMENSION PR(50), PRN(50), P(50), A(50,51) , U(50), UN(50)
    1 READ (5,4)   N
      READ (5,4) NX1
      READ (5,3) (X(I),  I =1,N)
      READ (5,3) (X1(I), I =1,NX1)
      READ (5,3)  (PR(I), I = 1, N)
      READ (5,3) (PRN(I),  I = 1,N)
      READ (5,6) N1, N2, N3, N4, N5, N6
    6 FORMAT (6A6)
      U(1) = 1.
      DO 10 J = 1, NX1
      LLL(J) = LOOK (X,X1,J)
      IF(LLL(J).GE.N) GO TO 20
   10 CONTINUE
      GO TO 25
   20 NX1 = J - 1
   25 LL = 2*N + NX1
      DO 50 I = 1, LL
      DO 50 J = 1, LL
   50 A(I,J) = 0.0
      DO 100 K = 2,N
      KI = K - 1
      A(KI,KI)= PR(K)*PR(K) + PRN(K)*PRN(K)
      KK = K + N - 2
      A(KI,KK) = (1. - PR(K))*PR(K)
      KKK = KK + 1
  100 A(KI,KKK) = (1. - PRN(K))*PRN(K)
      DO 200  K =    1,N
      KK = K + N - 1
      KI = K + 1
      A(KK,K) = (PR(KI))*(1. - PR(KI))
      KKK = 2*N + K - 1
      KN = N - 1
      IF (K.GT.KN) GO TO 40
      A(KK,KK) = A(KK,KK) + (1. - PR(KI))**2
   40 IF(K.GT.NX1) GO TO 41
      A(KK,KK) = A(KK,KK) + 1.
   41 A(KK,KK) = A(KK,KK) + (1. - PRN(K))**2
      IF (K.LT.2) GO TO 30
      KKKK = K - 1
      A(KK,KKKK) = PRN(K)* (1. - PRN(K))
      GO TO 200
   30 A(N,LL) =-(PRN(1))*(1. - PRN(1))
  200 CONTINUE
```

```
      DO 300  K = 1, NX1
      KK = 2 * N + K - 1
      A(KK,KK) = 1.
      KKK = N + K - 1
  300 A(KK,KKK) = 1.
      DO 450  K = 1,NX1
      KK = K + N - 1
      KKK = 2*N + K - 1
  450 A(KK,KKK) = 1.
      DO 299 J =1, NX1
      LJ = LLL(J)
      L1 = LJ - 1
      K = LJ + 1
      IF (LJ.GT.N) GO TO 333
      DELX = X(LJ + 1) - X(LJ)
      DX = (X(K) - X1(J))/DELX
      DXX = (X1(J) - X(LJ))/DELX
      NNJ = 2*N + J - 1
      IF (LJ.EQ.1) GO TO 135
      A(L1,L1) = A(L1,L1) + DX**2
      A(L1,LJ) = A(L1,LJ) + DX*DXX
      A(L1,NNJ) = A(L1,NNJ) - DX
      A(NNJ,L1) = A(NNJ,L1) - DX
      A(LJ,L1) = A(LJ,L1) + DX*DXX
  149 A(LJ,LJ) = A(LJ,LJ) + DXX**2
      A(LJ,NNJ) = A(LJ,NNJ) - DXX
      A(NNJ,LJ) = A(NNJ,LJ) - DXX
      A(NNJ,NNJ) = A(NNJ,NNJ) + 1.
      GO TO 299
  135 A(1,LL) = A(1,LL) -DXX*DX
      A(NNJ,LL) = A(NNJ,LL) + DX
      GO TO 149
  299 CONTINUE
  333 DO 700  I = 1,LL
      WRITE (6,900)   (A(I,J), J = 1,12)
  700 CONTINUE
      WRITE (6,2)
    2 FORMAT (1H1)
      IF (LL - 13) 33,33,34
   34 DO 800  I = 1,LL
      WRITE (6,900 )  (A(I,J),  J = 13,24)
  800 CONTINUE
      WRITE (6,2)
      IF (LL - 25) 33, 33, 35
   35 DO 1105 I = 1,LL
      WRITE (6,900 )  (A(I,J),  J = 25,36)
 1105 CONTINUE
   33 NP1 = 2*N + NX1 - 1
  449 CALL EMSLVR (A,COE,1,NP1,1)
```

```
      DO 400  K = 2, N
      KK = K - 1
  400 U(K) = COE(KK)
      DO 500  K = 1, N
      KKK = N + K - 1
  500 UN(K) = COE(KKK)
      DO 600  K = 1,NX1
      L = 2 *N + K - 1
  600 UNX1(K) = COE(L)
      WRITE (7,8) N1, N2, N3, N6
    8 FORMAT (36X,3A6,20X,A6)
      WRITE (6,1100)  (U(K), K = 1,N)
      WRITE (6,1100)  (UN(K), K = 1,N)
      WRITE (6,1100) (UNX1(K) , K = 1,NX1)
      WRITE (7,1100)  (U(K), K = 1,N)
      WRITE (7,1100)  (UN(K), K = 1,N)
      WRITE (7,1100) (UNX1(K) , K = 1,NX1)
 1100 FORMAT (1X,  6(F8.2,2X))
  900 FORMAT (1X, 12F10.7)
      GO TO 1
    3 FORMAT (F10.3)
    4 FORMAT (I2)
      END


      DOUBLE PRECISION FUNCTION LOOK (X,X1,J)
      DOUBLE PRECISION X,X1
      DIMENSION X(50), X1(50)
      DO  10  L = 1,100
      IF (X(L).GT.X1(J)) GO TO  20
   10 CONTINUE
   20 LOOK = L - 1
      RETURN
      END
```

```
      SUBROUTINE EMSLVR (A,ACOE,MORDER,N,MPTO)

C     WILL ORDER THE MATRIX BEFORE EACH ELIMINATION IF MORDER=+1
C     N= ORDER OF MATRIX
C     WILL SOLVE AN (N)X(N+1) MATRIX
C     REQUIRES MATRICES OF THE FORM (A)X(COE)=(B)
C     ACOE=VARIABLES TO BE SOLVED FOR
C     A(I,J)= MATRIX ENTRIES
C     COLUMN (I,N+1) OF A MATRIX CORRESPONDS TO COLUMN MATRIX B
C     DIMENSIONED VARIABLES MUST BE AT LEAST OF ORDER N OR N+1
C     DIMENSION A(N,N+1), IC(N), COE(N+1), ACOE(N)
C     ANSWERS TO SINGULAR MATRICES ARE ZERO(0)
      DOUBLE PRECISION ACOE,A,COE,AX,AY,SUM
      DIMENSION ACOE(50), A(50,51), IC(50), COE(51)
      NM=N
      NN=0
      KK=0
      MM=0
      NP1=N+1
      NM1=N-1
      DO 3 J=1,N
    3 A(J,NP1)=-A(J,NP1)
C     INITIALIZE SUBSCRIPT COLUMN
  799 DO 800 J=1,N
  800 IC(J)=J
      KKK=0
      IF(MORDER)999,75,999
C     - - - - - - - - - - - - - - - - - - - - - -
C     MATRIX ORDERING ROUTINE
C     - - - - - - - - - - - - - - - - - - - - - -
  999 KKK=KKK+1
      AB =DABS(A(KKK,KKK))
      IBIG=KKK
      JBIG=KKK
      DO 901 I=KKK,N
      DO 901 J=KKK,N
      IF(AB-DABS(A(I,J)))900,901,901
  900 AB =DABS(A(I,J))
      IBIG=I
      JBIG=J
  901 CONTINUE
  910 DO 920 I=1, NP1
      AX=A(KKK,I)
      A(KKK,I)=A(IBIG,I)
  920 A(IBIG,I)=AX
      DO 930 J=1,N
      AY=A(J,KKK)
      A(J,KKK)=A(J,JBIG)
  930 A(J,JBIG)=AY
```

```
  940 IDUM=IC(KKK)
      IC(KKK)=IC(JBIG)
      IC(JBIG)=IDUM
      IF(NM1-KKK) 71,71,999
C     --------------------------------------
   71 CONTINUE
   75 NN=NN+1
      NNN=NN+1
      MM=MM+1
C     ----------------------------------
C     CHECK FOR SINGULAR MATRIX
C     ----------------------------------
      IF (A(NN,NN)) 77,1700,77
C     ----------------------------------
C     MATRIX SOLUTION ROUTINE
C     --------------------------------------
   77 DO 80 I=NN,N
      IF(A(I,NN)) 79,80,79
   79 DO 80 J=NNN,NP1
      A(I,J)=A(I,J)/A(I,NN)
   80 CONTINUE
      KK=KK+1
      IF(KK-NM1)85,85,100
   85 DO 90 I=NNN,N
      IF(A(I,NN)) 89,90,89
   89 DO 90 J=NNN,NP1
      A(I,J)=A(I,J)-A(NN,J)
   90 CONTINUE
      IF(MORDER)91,75,91
   91 IF(KK-NM1+1)92,92,75
   92 KKK=MM
      GO TO 999
C     ----------------------------------
C     BACK SOLVE UPPER TRIANGULAR MATRIX
C     ----------------------------------
  100 COE(NP1)=1.0
      DO 110 K=1,NM
      SUM=0.0
      J=NP1-K
      L=J+1
      DO 109 I=L,NP1
  109 SUM=SUM-A(J,I)*COE(I)
  110 COE(J)=SUM
C     ----------------------------------
C     REORDER ANSWER MATRIX
C     ----------------------------------
      DO 1005 I=1,NM
      K1=IC(I)
```

```
      ACOE(K1)=COE(I)
 1005 CONTINUE
      IF(MPTO)1500,1600,1500
 1500 CONTINUE
      WRITE (6,2)
      DO 1599 J=1,NM
      WRITE (6,1) ACOE(J), J, IC(J)
 1599 CONTINUE
 1600  RETURN
    1 FORMAT(1X,1H E15.6,2I8)
    2 FORMAT(1X,1H041H VARIABLE          IDENT                          )
 1700 PRINT 10
   10 FORMAT (1X,1H0,16H SINGULAR MATRIX)
      DO 1900 I=1,N
 1900 ACOE(I)=0.
      RETURN
      END
```

APPENDIX III

FORTRAN PROGRAM FOR TWENTY QUESTION METHOD

Appendix III

COMPUTER PROGRAM TO CALCULATE COEFFICIENTS FOR A UTILITY CURVE

Purpose

The purpose of this program is to calculate the coefficients for a utility curve from the utile-dollar data points computed by one of the data smoothing programs.

Language

Fortran IV (IBM 7040 Computer).

Symbolic Dictionary

VARIABLE	S/A*	I/O**	DESCRIPTION
N	S	I	Total number of data points.
X	A	I&O	Dollar values for the data points.
U	A	I&O	Utility values for the data points.
B	S	O	The unknown variable within the logarithm function which causes the curve to go through the (0,0) point. Initially it is a guess to start processing. Thereafter it is computed internally.
BG	S	-	That value of the variable B whose first derivative is greater than zero.
BL	S	-	That value of the variable B whose first derivative is less than zero.
BM	S	-	That value of the variable B whose first derivative is equal to zero or within limits set by the program.

*S - Single variable; A - Array of variables.

**I - Input; O - Output.

UCAL	A	O	Calculated utility value for the data points. Computed internally with the use of the computed coefficients.
DEVIAT	A	O	Deviation of data point utility and the calculated utility values.
STDEV	S	O	Standard deviation of the data point utility and the calculated utility values.
A	S	O	The unknown variable which controls the magnitude of the curve. This is computed internally.
MN	S	I	Number of iterations in the process of calculating BG's and BL's to determine BM.
MTEST	S	I	Total number of X values dropped.
NBTEST	S	I	Number of iterations to find an FOFB less than zero.

Program Routine

In this method of iteration, two values of B are determined so that the root of the equation lies between them. Since the root lies between the two numbers, the graph of FOFB = f(B) must cross the x-axis between $B = B_1$ and $B = B_2$, and the $FOFB_1$ and $FOFB_2$ must have opposite signs. We designate the value of B which gives FOFB less than zero as BL, while the value of B which gives FOFB greater than zero as BG. Having established these two limiting points, BL and BG, we used 20 iterations to calculate the correct value of B (designated BM) where:

$$BM = (BG + BL)/2.$$

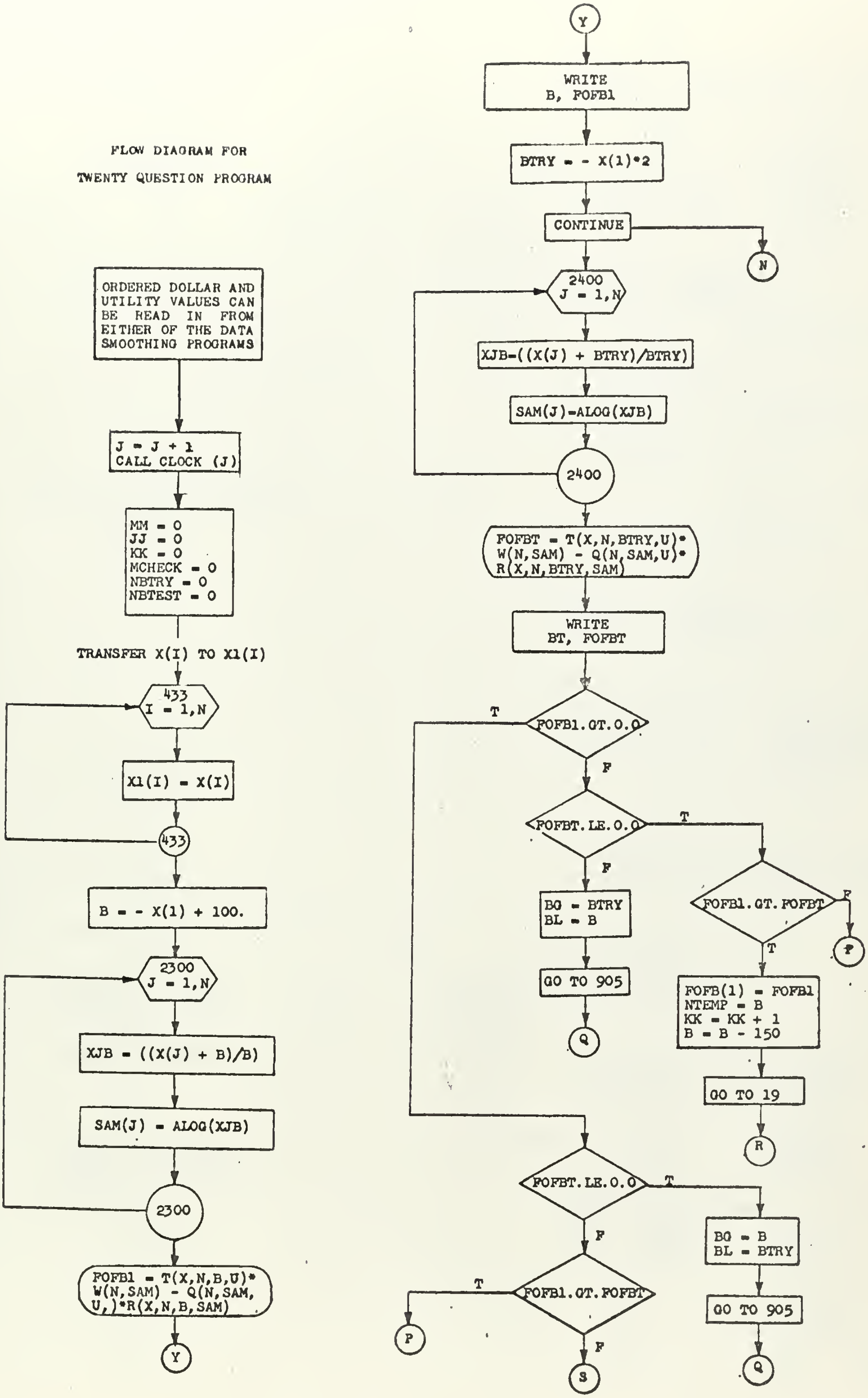
The method of determining the FOFB follows the general mathematical model described in Chapter III. In brief, the

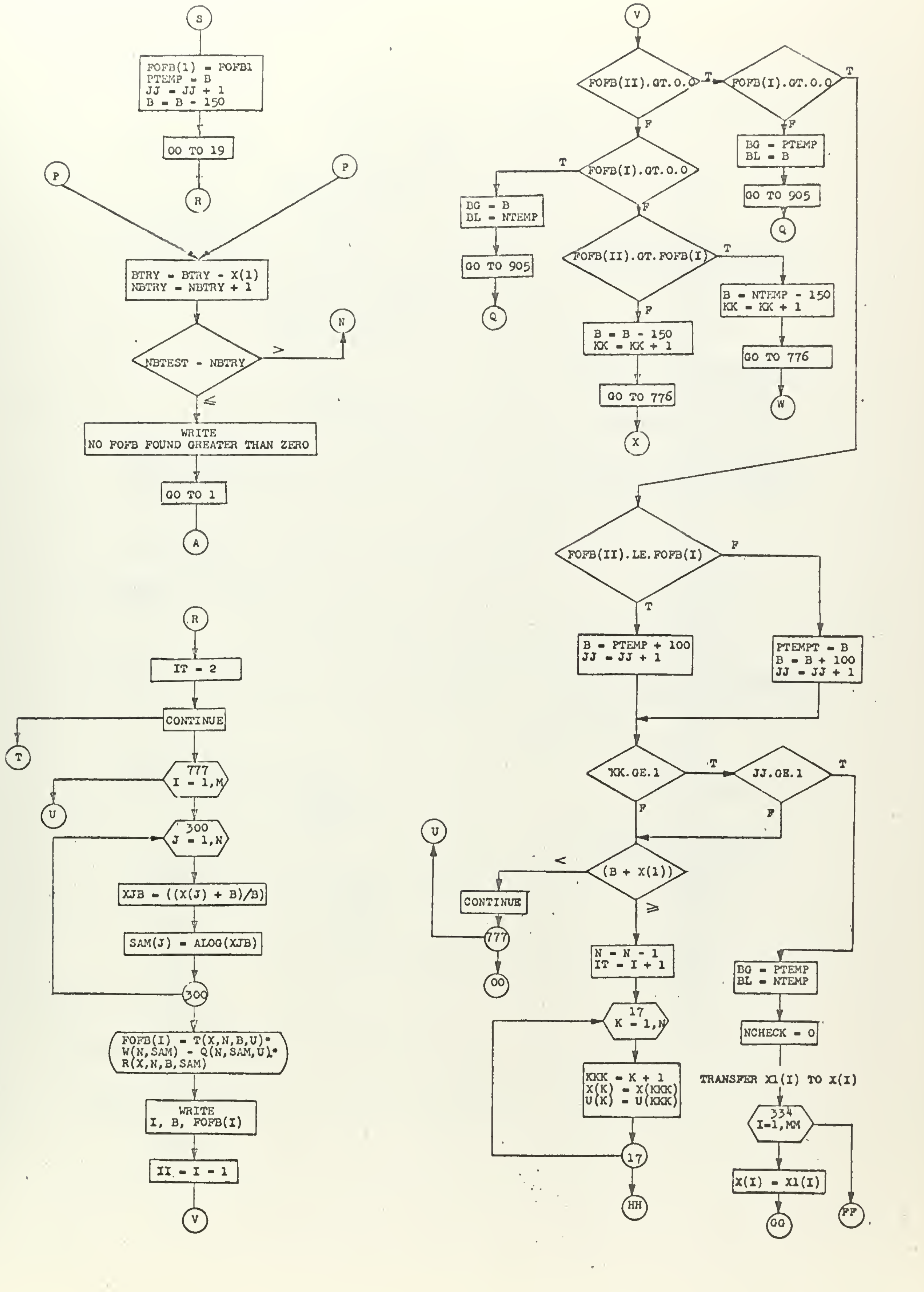
general mathematical equation was set equal to e_j. Both sides of the equation were squared and the partial derivatives of E with respect to a and b were set equal to zero in order to minimize the total error. Several collective terms which were of a repetitive nature were designated by alphabetic titles and included in the program as function programs. This allowed FOFB to be identified in the mainline as a mathematical expression consisting of the several functions.

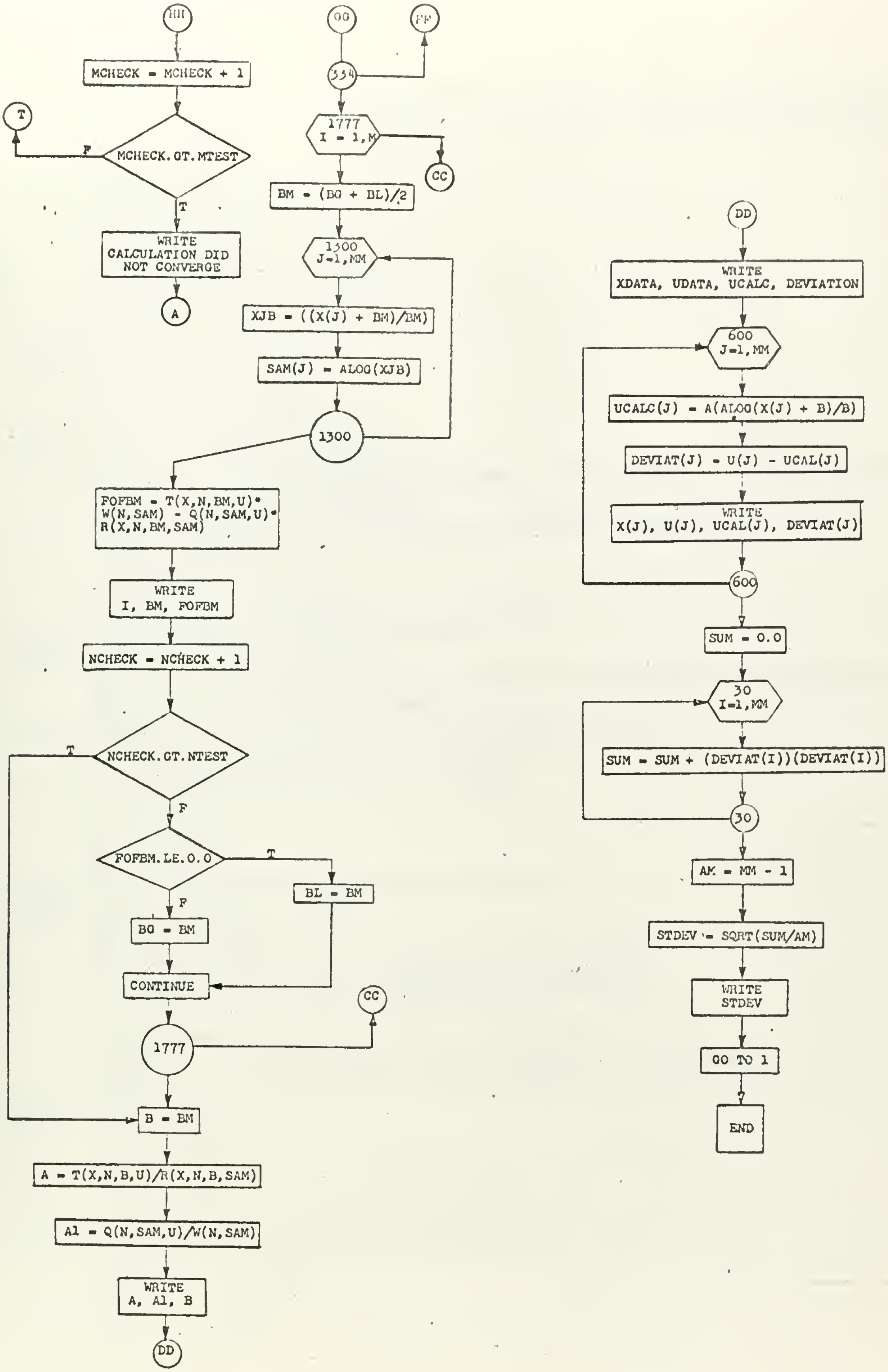
When BM is determined, this value is used to calculate the other unknown variable A. The program calculates the utility values for the dollar points using the calculated coefficients. These values are used to determine the standard deviation for the calculated utility curve.

FLOW DIAGRAM FOR

TWENTY QUESTION PROGRAM




S
FOFB(1) = FOFB1
PTEMP = B
JJ = JJ + 1
B = B - 150
GO TO 19
R
P
P
BTRY = BTRY - X(1)
NBTRY = NBTRY + 1
NBTEST - NBTRY
>
N
≤
WRITE
NO FOFB FOUND GREATER THAN ZERO
GO TO 1
A
R
IT = 2
CONTINUE
T
777
I = 1,M
U
300
J = 1,N
XJB = ((X(J) + B)/B)
SAM(J) = ALOG(XJB)
300
FOFB(I) = T(X,N,B,U)*
W(N,SAM) - Q(N,SAM,U)*
R(X,N,B,SAM)
WRITE
I, B, FOFB(I)
II = I - 1
V
V
FOFB(II).GT.0.0
T
FOFB(I).GT.0.0
T
F
F
BG = PTEMP
BL = B
GO TO 905
Q
T
FOFB(I).GT.0.0
BG = B
BL = NTEMP
GO TO 905
Q
F
FOFB(II).GT.FOFB(I)
T
F
B = NTEMP - 150
KK = KK + 1
GO TO 776
W
B = B - 150
KK = KK + 1
GO TO 776
X
FOFB(II).LE.FOFB(I)
F
T
B = PTEMP + 100
JJ = JJ + 1
PTEMPT = B
B = B + 100
JJ = JJ + 1
KK.GE.1
T
JJ.GE.1
T
F
F
U
<
(B + X(1))
≥
CONTINUE
777
OO
N = N - 1
IT = I + 1
17
K = 1,N
KKK = K + 1
X(K) = X(KKK)
U(K) = U(KKK)
17
HH
BG = PTEMP
BL = NTEMP
NCHECK = 0
TRANSFER X1(I) TO X(I)
334
I=1,MM
X(I) = X1(I)
GG
FF



HH
GG
FF
MCHECK = MCHECK + 1
T
F
MCHECK.GT.MTEST
T
WRITE CALCULATION DID NOT CONVERGE
A
334
1777 I = 1,M
CC
BM = (BG + BL)/2
1300 J=1,MM
XJB = ((X(J) + BM)/BM)
SAM(J) = ALOG(XJB)
1300
FOFBM = T(X,N,BM,U)*W(N,SAM) - Q(N,SAM,U)*R(X,N,BM,SAM)
WRITE I, BM, FOFBM
NCHECK = NCHECK + 1
T
NCHECK.GT.NTEST
F
FOFBM.LE.0.0
T
BL = BM
F
BG = BM
CONTINUE
CC
1777
B = BM
A = T(X,N,B,U)/R(X,N,B,SAM)
A1 = Q(N,SAM,U)/W(N,SAM)
WRITE A, A1, B
DD
DD
WRITE XDATA, UDATA, UCALC, DEVIATION
600 J=1,MM
UCALC(J) = A(ALOG(X(J) + B)/B)
DEVIAT(J) = U(J) - UCAL(J)
WRITE X(J), U(J), UCAL(J), DEVIAT(J)
600
SUM = 0.0
30 I=1,MM
SUM = SUM + (DEVIAT(I))(DEVIAT(I))
30
AM = MM - 1
STDEV = SQRT(SUM/AM)
WRITE STDEV
GO TO 1
END

```
C     THIS SECTION OF THE PROGRAM IS THE TWENTY QUESTION METHOD
C     OF REGRESSION ANALYSIS.  IT FOLLOWS THE ORDERING SECTION.


      MCHECK = 0
      NBTRY = 0
C     ----------------------
C     TRANFER X(I) TO X1(I)
C     ----------------------
      DO 433  I = 1,N
  433 X1(I) = X(I)
   18 B = - X(1) + 100.
      DO 2300 J = 1,N
      XJB = ((X(J) + B)/B)
 2300 SAM(J) = ALOG (XJB)
      FOFB1    = T(X,N,B,U)*W(N,SAM) - Q(N,SAM,U)*R(X,N,B,SAM)
      WRITE (6,11)  B, FOFB1
   11 FORMAT (1X,4H B =,F10.3,10X,7HFOFB1 =,E15.6)
      BTRY = 15000.
   13 CONTINUE
      DO 2400  J = 1,N
      XJB = ((X(J) + BTRY)/BTRY)
 2400 SAM(J) = ALOG (XJB)
      FOFBT = T(X,N,BTRY,U)*W(N,SAM) - Q(N,SAM,U)*R(X,N,BTRY,SAM)
      WRITE (6,12) BTRY,FOFBT
   12 FORMAT (4H BT=,F10.3,10X,7HFOFBT =,E15.6)
      IF (FOFB1.GT.0.0) GO TO 3400
      IF (FOFBT.LE.0.0) GO TO 3300
      BG = BTRY
      BL = B
      GO TO 905
 3300 IF (FOFB1.GT.FOFBT) GO TO 3600
 3301 BTRY = BTRY - X(1)
      NBTRY = NBTRY + 1
      IF (ABS(FOFBT).LE..00001) GO TO 3302
      IF (ABS(FOFB1).LE..00001) GO TO 3303
      IF (NBTEST - NBTRY) 15, 15, 13
   15 WRITE (6,9)
    9 FORMAT (32H NO FOFB FOUND GREATER THAN ZERO)
      GO TO 1
 3302 BM = BTRY
      GO TO 1800
 3303 BM = B
      GO TO 1800
 3400 IF (FOFBT.LE.0.0) GO TO 3500
      IF (FOFB1.GT.FOFBT) GO TO 3301
      FOFB(1) = FOFB1
      PTEMP = B
      B = B - 150.
      JJ = JJ + 1
```

```
      GO TO 19
 3500 BG = B
      BL = BTRY
      GO TO 905
 3600 FOFB(1) = FOFB1
      NTEMP = B
      B = B - 150.
      KK = KK + 1
      GO TO 19
   19 IT = 2
   20 CONTINUE
      DO 777  I = IT, M
      DO 300  J = 1,N
      XJB = ((X(J) + B)/B)
  300 SAM(J) = ALOG(XJB)
      FOFB(I) = T(X,N,B,U)*W(N,SAM) - Q(N,SAM,U)*R(X,N,B,SAM)
      WRITE (6,16) I,B,FOFB(I)
   16 FORMAT(3H I=,I2,10X,3HB =,F10.3,10X,6HFOFB =,E15.6)
      II = I - 1
      IF (FOFB(II).GT.0.0) GO TO 720
      IF (FOFB(I).GT.0.0)  GO TO 200
      IF (FOFB(II).GT.FOFB(I)) GO TO 710
      B = B - 150.
      KK = KK + 1
      GO TO 776
  710 B = NTEMP - 150
      KK = KK + 1
      GO TO 776
  200 BG = B
      BL = NTEMP
      GO TO 905
  720 IF (FOFB(I).GT.0.0)  GO TO 201
      BL = B
      BG = PTEMP
      GO TO 905
  201 IF (FOFB(II).LE.FOFB(I)) GO TO 203
      PTEMP = B
      B = B + 100.
      JJ = JJ + 1
      GO TO 776
  203 B = PTEMP + 100.
      JJ = JJ + 1
  776 IF (KK.GE.1) GO TO 820
  778 IF (B + X(1))  810, 810, 777
      GO TO 777
  820 IF (JJ.GE.1) GO TO 920
      GO TO 778
  777 CONTINUE
  780 WRITE (6,790)
  790 FORMAT(30H CALCULATIONS DID NOT CONVERGE)
```

```
      GO TO 1
  810 N = N-1
      IT = I + 1
      DO 17  K = 1,N
      KKK = K + 1
      X(K) = X(KKK)
   17 U(K) = U(KKK)
      MCHECK = MCHECK + 1
      IF (MCHECK.GT.MTEST) GO TO 780
      GO TO 20
  920 BG = PTEMP
      BL = NTEMP
  905 NCHECK = 0
C     -----------------------
C     TRANSFER X1(I) TO X(I)
C     -----------------------
      DO 334  I = 1,MM
  334 X(I) = X1(I)
      DO 1777  I = 1, M
  910 BM = (BG + BL)/2.
      DO 1300  J = 1, MM
      XJB = ((X(J) + BM)/BM)
 1300 SAM(J) = ALOG (XJB)
      FOFBM = T(X,N,BM,U)*W(N,SAM) - Q(N,SAM,U)*R(X,N,BM,SAM)
      WRITE (6,14) I, BM, FOFBM
   14 FORMAT (3H I=,I2,10X,4HBM =,F10.3,10X,7HFOFBM =,E15.6)
      IF (ABS(FOFBM).LE..000001) GO TO 1800
      NCHECK = NCHECK + 1
      IF (NCHECK.GT.NTEST) GO TO 1800
      IF (FOFBM.LE.0.0) GO TO 1000
  999 BG = BM
      GO TO 1777
 1000 BL = BM
 1777 CONTINUE
 1800 B = BM
  500 A = T(X,N,B,U) / R(X,N,B,SAM)
      A1 = Q(N,SAM,U) / W(N,SAM)
      WRITE ( 6,700)
      WRITE (6,800) A, A1, B
      WRITE (6,151) N1, N2, N3
  151 FORMAT (/36X,3A6/)
      WRITE ( 6,850)
      DO 600   J = 1, MM
      UCAL(J) = A*(ALOG((X(J) +B)/B))
      DEVIAT(J) = U(J) - UCAL(J)
  600 WRITE (6,900) X(J), U(J), UCAL(J), DEVIAT (J)
      SUM = 0.0
      DO 30  I = 1, MM
   30 SUM = SUM + (DEVIAT(I))*(DEVIAT(I))
      AM = MM - 1
```

```
      STDEV = SQRT (SUM/AM)
      WRITE (6,31)  STDEV
   31 FORMAT(22H  STANDARD DEVIATION =,F10.5)
  700 FORMAT (1H1,31X,18H UTILITY CURVE FIT//)
  800 FORMAT (7X,3H A=,F10.3,10X,4H A1=,F10.3,10X,3H B=,F10.3)
  850 FORMAT (7X, 5HXDATA, 15X,5HUDATA, 15X,5HUCALC, 12X,
     19HDEVIATION)
      GO TO 1
      WRITE (6,2)
    2 FORMAT (1H1)
  900 FORMAT (4X, 3(F10.3,10X), E12.6)
      END


      FUNCTION Q ( N, SAM, U)
      DIMENSION SAM (20), U (20)
      Q = 0.0
      DO 10  J =  1, N
   10   Q = Q  + U(J) * SAM (J)
      RETURN
      END


      FUNCTION R ( X , N, B, SAM)
      DIMENSION X (20), SAM (20)
      R = 0.0
      DO 10  J =  1, N
   10 R = R + SAM(J) * ( 1. / (X(J) + B)-1./B)
      RETURN
      END


      FUNCTION T ( X, N, B, U)
      DIMENSION X (20),  U (20)
      T  = 0.0
      DO 10  J =  1, N
   10  T = T + U(J) * ( 1./(X(J) + B) -1. / B )
      RETURN
      END


      FUNCTION W ( N, SAM)
      DIMENSION SAM (20)
      W = 0.0
      DO 10  J =  1, N
   10   W  = W  + SAM(J)  **2
      RETURN
      END
```

APPENDIX IV

FORTRAN PROGRAM FOR STEP-BY-STEP METHOD

Appendix IV

COMPUTER PROGRAM TO CALCULATE UTILITY VALUES BY THE STEP-BY-STEP PROCEDURE

Purpose

The purpose of this program is to incorporate the Part II data points with the Part I data points by means of a data smoothing process.

Language

Fortran IV (IBM 7040 Computer).

Symbolic Dictionary

VARIABLE	S/A*	I/O**	DESCRIPTION
M	S	I	Total number of Part I data points.
NX1	S	I	Total number of Part II data points.
P	A	I&O	Dollar values from questionnaire entries which become the dollar values for the positive Part I points.
PR	A	I&O	Probabilities used to determine a positive utility value from a negative utility value.
PRN	A	I&O	Probabilities used to determine a negative utility value from a positive utility value.
X1	A	I	Dollar values for the Part II data points.

*S - Single variable; A - Array of variables.

**I - Input; O - Output.

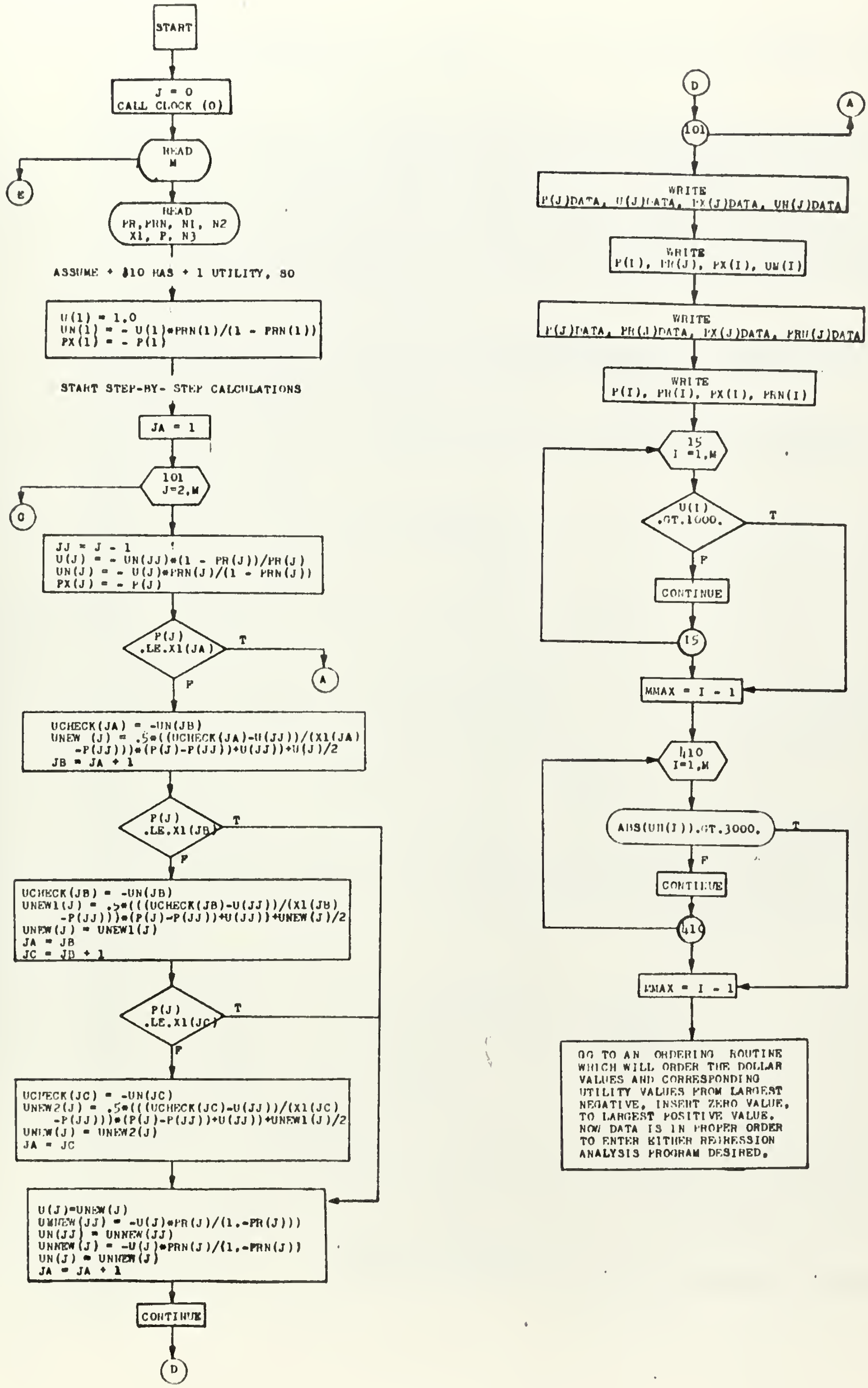
PX	A	-	Dollar values for negative Part I points.
U	A	0	Positive utility values for Part I data points.
UN	A	0	Utility value for the negative Part I data points.

Program Routine

This program utilizes probabilities derived from a utility questionnaire to compute utility values corresponding to the dollar values of the data points. The procedure used to calculate utility values is called the step-by-step procedure from the method of incorporating the Part II data. The utility values are determined for Part I points using Equation (8). After each value for Part I is calculated, the program tests to see if the positive dollar value is greater than the first Part II point. When this is found to exist, the normal procedure is terminated temporarily to incorporate the Part II point. The function is considered linear between the two bracketing Part I points and the Part II point. A mean value for the upper Part I point is calculated. This mean value is used to continue the process. After all calculations, all dollar values are transferred to one array and ordered ascending from the largest negative value. Corresponding utility values are ordered accordingly.

All dollar values must be transferred to one array and ordered from largest negative to largest positive before this data can be read into either of the regression analysis programs. The corresponding utility values are ordered accordingly.

FLOW DIAGRAM FOR STEP-STEP-CALCULATIONS



```
C     THIS SECTION OF THE PROGRAM IS THE STEP-BY-STEP METHOD
C     OF DATA SMOOTHING.

C     PROGRAM DESIGNED TO DERIVE UTILES FROM QUESTIONNAIRE
C     THE P(I)S ARE DERIVED FROM PART I OF QUESTIONNAIRE
C     P(1) MUST BE THE LARGEST NEGATIVE DOLLAR VALUE
C     THE X1(I)S ARE DOLLAR VALUES FROM PART II OF QUESTIONNAIRE
C     PR VALUES ARE PROBABILITIES FOR THE 2 TO 1 RATIO QUESTIONS
C     PRN VALUES ARE PROBABILITIES OF THE EQUAL RATIO QUESTIONS
C     M = NUMBER OF DATA POINTS
      DIMENSION X1(20), P(20), SAM(20)
      DIMENSION UCHECK(20),UNEW(20),UNNEW(20) ,UNEW1(20),UNEW2(20)
      DIMENSION U(20), UN(20), PR(20), PRN(20)
      DIMENSION PX(20), PTEMP(20)
      DIMENSION UCAL(20), DEVIAT(20), UUTEMP(20), X(20)
   11 READ (5,4) M
      READ (5,4) NX1
      READ (5,3)  (P(I), I =1,M)
      READ (5,3) (X1(I), I =1,NX1)
      READ (5,3)  (PR(I),  I = 1,M)
      READ (5,3) (PRN(I), I = 1,M )
      READ (5,150)  N1, N2, N3, N4, N5, N6
  150 FORMAT (6A6)
C     --------------------------------
C     ASSUME +$10 HAS +1 UTILITY, SO
C     DETERMINE UTILITY FOR -$10 BY
C     --------------------------------
      U(1) = 1.0
      UN(1) =-U(1) * PRN(1) / (1. - PRN(1))
      PX(1) = -P(1)
C     -----------------------------------------------------
C     START DO LOOP FOR (1) +X TO -2X AND (2) +2X TO - X
C     -----------------------------------------------------
      JA = 1
      DO 101 J = 2, M
      JJ = J - 1
      U(J) = -UN(JJ)  * (1. - PR(J)) / PR(J)
      UN(J) = -U(J)   * PRN(J) / (1. - PRN(J))
      PX(J) = -P(J)
      IF (JA.GT.NX1) GO TO 107
      IF (P(J).LT.X1(JA)) GO TO 107
      UCHECK(JA) = -UN(JA)
      UNEW(J)=.5*(((UCHECK(JA)-U(JJ))/(X1(JA)-P(JJ)))*(P(J)-
     1P(JJ)) + U(JJ)) + U(J)/2.
     1U(JJ)) + U(J)/2.
      WRITE (6,12)    JA,U(J),UNEW(J)
   12 FORMAT(5H0JA =,I2,10X,8H U(J)  =,F15.3,10X,11H UNEW(J)  =,
     1F15.3)
```

```
      JB = JA + 1
      IF (JB.GT.NX1) GO TO 102
      IF (P(J).LT.X1(JB)) GO TO 102
      UCHECK(JB) = -UN(JB)
      UNEW1(J) = .5*(((UCHECK(JB)-U(JJ))/(X1(JB)-P(JJ)))*(P(J)-
     1P(JJ)) + U(JJ)) + UNEW (J)/2.
      UNEW(J) = UNEW1(J)
      WRITE (6,302)   JB,U(J),UNEW(J)
  302 FORMAT (5H0JB =,I2,10X,8H U(J)  =,F15.3,10X,11H UNEW(J)  =,
     1F15.3)
      JA = JB
      JC = JB + 1
      IF (JC.GT.NX1) GO TO 102
      IF (P(J).LT.X1(JC)) GO TO 102
      UCHECK(JC) = -UN(JC)
      UNEW2(J) = .5*(((UCHECK(JC)-U(JJ))/(X1(JC)-P(JJ)))*(P(J)-
     1P(JJ)) + U(JJ)) + UNEW1(J)/2.
      UNEW(J) = UNEW2(J)
      WRITE (6,303)   JC,U(J),UNEW(J)
  303 FORMAT (5H0JC =,I2,10X,8H U(J)  =,F15.3,10X,11H UNEW(J)  =,
     1F15.3)
      JA = JC
  102 U(J) = UNEW(J)
      UNNEW(JJ) =-U(J)*(PR(J)/(1.-PR(J)))
      WRITE (6,13)    JJ,UN(JJ),UNNEW(JJ)
   13 FORMAT (5H0JJ =,I2,10X,8HUN(JJ) =,F15.3,10X,11HUNNEW(JJ) =,
     1F15.3)
      UN(JJ) = UNNEW(JJ)
      UNNEW(J) = -U(J)*PRN(J)/(1. - PRN(J))
      WRITE (6,14)   J, UN(J), UNNEW(J)
   14 FORMAT(5H0 J =,I2,10X,8HUN(J)  =,F15.3,10X,11HUNNEW(J)  =,
     1F15.3)
      UN(J) = UNNEW(J)
      JA = JA + 1
      GO TO 101
  107 WRITE (6,16) J,U(J)
   16 FORMAT (5H0 J =,I2,10X,8H U(J)  =,F15.3)
  101 CONTINUE
      WRITE (6,151) N1, N2, N3
  151 FORMAT (/36X,3A6/)
      WRITE (6,901)
  901 FORMAT(/12X,8HP(J)DATA,12X,8HU(J)DATA,11X,9HPX(J)DATA,
     111X,9HUN(J)DATA/)
     19HUN(J)DATA/)
      WRITE (6,1000) (P(I), U(I), PX(I),   UN(I), I = 1,M)
 1000 FORMAT (4(5X,F15.3))
      WRITE (6,3001)
 3001 FORMAT (/12X,8HP(J)DATA,12X,9HPR(J)DATA,10X,9HPX(J)DATA,
     110X,10HPRN(J)DATA/)
```

```
      WRITE (6,1000) (P(I),PR(I),PX(I),PRN(I), I =1,M)
    3 FORMAT (F10.5)
    4 FORMAT (I2)
      WRITE (7,152) N1, N2, N3, N6
  152 FORMAT (36X,3A6,20X,A6)
      WRITE (7,1100) (U(K), K =1, M)
      WRITE (7,1100)(UN(K), K =1, M)
 1100 FORMAT (1X,6(F10.4,3X))
      WRITE (6,2)
    2 FORMAT (1H1)
      GO TO 11
      END
```

APPENDIX V

FORTRAN PROGRAM FOR ORIGINAL LOGARITHM METHOD

Appendix V

COMPUTER PROGRAM TO CALCULATE COEFFICIENTS FOR A UTILITY CURVE

Purpose

The purpose of this program is to calculate the coefficients for a utility curve from the utile-dollar data points computed by one of the data smoothing programs.

Language

Fortran IV (IBM 7040 Computer).

Symbolic Dictionary

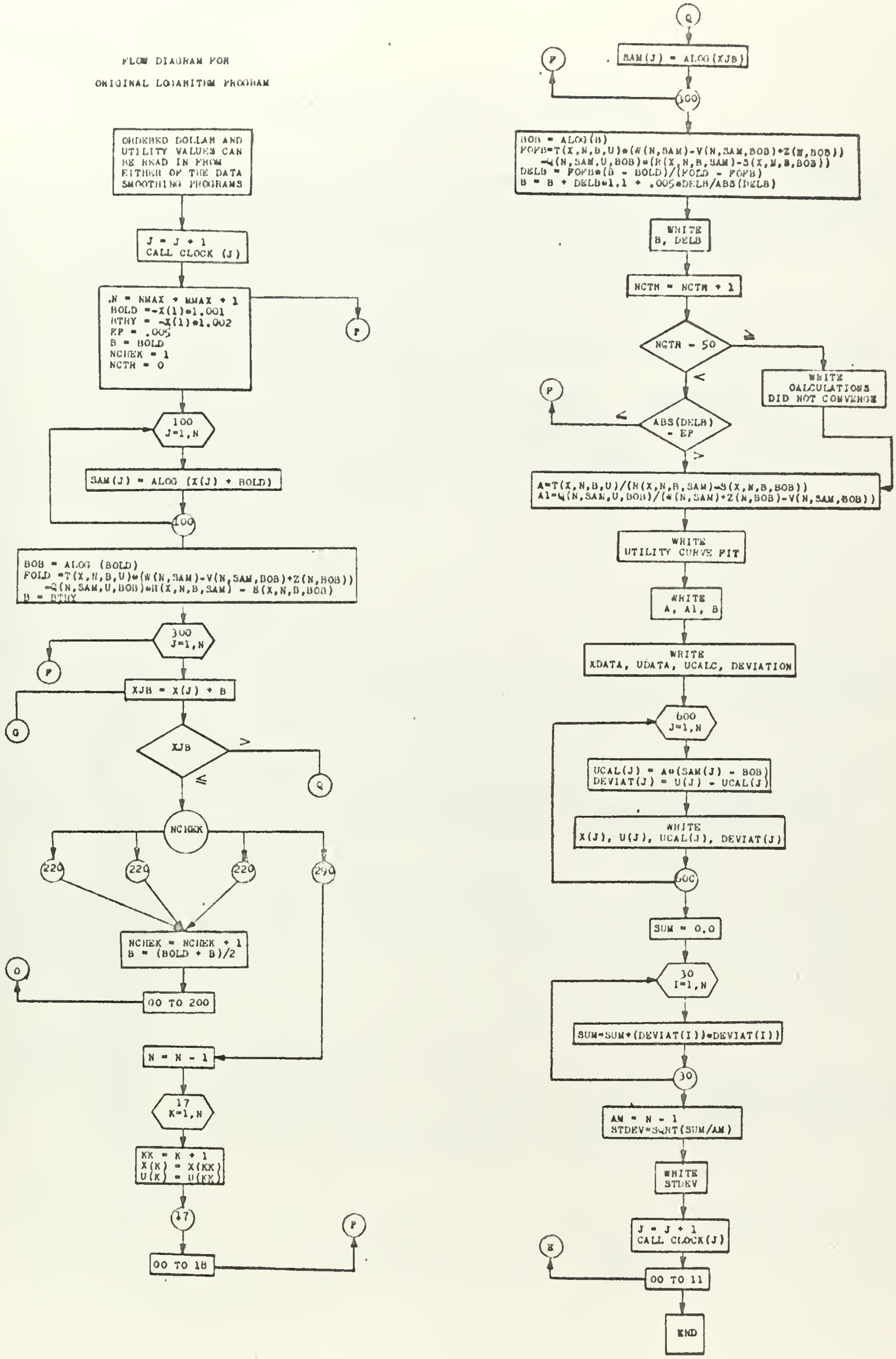
VARIABLE	S/A*	I/O**	DESCRIPTION
N	S	I	Total number of data points.
U	A	I&O	Utility values for the data points.
X	A	I&O	Dollar values for the data points.
B	S	O	The unknown variable within the logarithm function which causes the curve to go through the (0,0) point.
UCAL	A	O	Calculated utility value for the data points. Computed internally with the use of the computed coefficients.
DEVIAT	A	O	Deviation of data point utility and the calculated utility values.
STDEV	S	O	Standard deviation of the data point utility and the calculated utility values.
A	S	O	The unknown variable which controls the magnitude of the curve. This is computed internally.

*S - Single variable; A - Array of variables.

**I - Input; O - Output.

FLOW DIAGRAM FOR

ORIGINAL LOGARITHM PROGRAM



```
C     THIS IS THE ORIGINAL LOGARITHM PROGRAM.

C     PROGRAM DESIGNED TO FIT AN EQUATION TO UTILITY CURVES
C     BASIC EQUATION. EPS = U(J) - A*ALOG(X(J)+B) - A*ALOG(B)
C     TO SIMPLY THE FORMULAS, THE FOLLOWING TERMS ARE SET
C     EQUAL TO REOCCURRING GROUPS OF VALUES.
C     BOB = ALOG(B)
C     SAM(J) = ALOG (X(J) + B)
C     H = ((1./X(J)+B)-1./B)
C     DELEPS/DELA=A*SUM(SAM(J)*SAM(J))-A*2.*SUM(SAM(J)*BOB)+
C     A*SUM(BOB*BOB)-SUM(SAM-BOB)
C     DELEPS/DELB =A*SUM((SAM(J)-BOB)*H) - SUM(U(J)*H)
C     TO SIMPLY CALCULATIONS, THE FOLLOWING TERMS ARE SET
C     EQUAL TO REOCCURRING GROUPS OF VALUES.
C     Q = U(J)*(SAM(J)-BOB)
C     R = SAM(J)*H
C     S = BOB*H
C     T = U(J)*H
C     V = 2.*SAM(J)*BOB
C     W = SAM(J)**2
C     Z = BOB*BOB
C     A = T/(R-S) = Q/(W-V+Z)
      DIMENSION X(20), SAM(20), U(20), UCAL(20), DEVIAT (20)
    2 READ (5,10) BTRY, BOLD, N, EP
      READ (5,20) (X(J) , U (J), J = 1,N)
      WRITE (6,2000)
      WRITE (6,2100) BTRY, BOLD, N, EP
 2000 FORMAT (7X,4HBTRY,15X,4HBOLD,15X,1HN,15X,2HEP///)
 2100 FORMAT (4X,F10.3,9X,F10.3,10X,I2,14X,F6.4///)
      WRITE (6,1111)
 1111 FORMAT (13X,1HB,15X,4HDELB//)
   10 FORMAT (2F10.3,I2, F6.4)
   20 FORMAT (2F10.3)
      B=BOLD
      NCHEK = 1
      NCTR = 0
      DO 100   J= 1,N
  100 SAM(J) = ALOG (X(J) + BOLD)
      BOB = ALOG (BOLD)
      FOLD = T(X,N,B,U) * ( W(N,SAM) - V(N,SAM,BOB) + Z(N,BOB))
     1 -Q(N,SAM,U,BOB)*R(X,N,B,SAM) - S(X,N,B,BOB)
      B = BTRY
  180 DO 300   J = 1, N
  200 XJB = X(J) + B
      IF (XJB)  210, 210, 300
  210 GO TO ( 220, 220, 220, 290)  ,   NCHEK
  220 NCHEK = NCHEK  + 1
      B = (BOLD + B)/2.
```

```
      GO TO 200
  290 B = - X(1)*1.001
      GO TO 500
  300 SAM(J) = ALOG (XJB)
      BOB = ALOG (B)
      FOFB = T(X,N,B,U) * ( W(N,SAM) - V(N,SAM,BOB) + Z(N,BOB))
     1 -Q(N,SAM,U,BOB)*R(X,N,B,SAM) - S(X,N,B,BOB)
      DELB = FOFB* (B - BOLD) / (FOLD - FOFB)
      B = B + DELB * 1.1 + .005 * DELB/ABS(DELB)
      WRITE (6,9981) B , DELB
 9981 FORMAT (7X,E10.3,10X,E10.3)
      NCTR = NCTR + 1
      IF ( NCTR - 50) 400, 450, 450
  400 IF ( ABS  (DELB) - EP) 500, 500, 180
  450 WRITE (6,3000)
 3000 FORMAT (15X,28HCALCULATION DID NOT CONVERGE//)
  500 A = T(X,N,B,U) / (R(X,N,B,SAM) - S(X,N,B,BOB))
      WRITE ( 6,700)
      WRITE (6,800) A,B
      WRITE ( 6,850)
      DO 600    J = 1, N
      UCAL(J) = A*(SAM(J) - BOB)
      DEVIAT(J) = U(J) - UCAL(J)
  600 WRITE (6,900) X(J), U(J), UCAL(J), DEVIAT (J)
      SUM = 0.0
      DO 30 J = 1,N
   30 SUM = SUM + (DEVIAT(J))*(DEVIAT(J))
      AM = N - 1
      STDEV = SQRT (SUM/AM)
      WRITE (6,31) STDEV
      GO TO 2
   31 FORMAT    (21H0STANDARD DEVIATION =,F10.5)
  700 FORMAT (1H1,31X,18H UTILITY CURVE FIT///)
  800 FORMAT(7X,3H A=,F10.3,10X,3H B=,F10.3//)
  850 FORMAT (11X,5HXDATA, 15X,5HUDATA, 15X,5HUCALC, 16X,
     19HDEVIATION/)
  900 FORMAT (4X,F12.3,10X,2(F10.3,10X),E15.6)
      END
```

```
      FUNCTION Q ( N, SAM, U)
      DIMENSION SAM (20), U (20)
      Q = 0.0
      DO 10  J =  1, N
10      Q = Q  + U(J) * SAM (J)
      RETURN
      END


      FUNCTION R ( X , N, B, SAM)
      DIMENSION X (20), SAM (20)
      R = 0.0
      DO 10  J =  1, N
10    R = R + SAM(J) * ( 1. / (X(J) + B)-1./B)
      RETURN
      END


      FUNCTION T ( X, N, B, U)
      DIMENSION X (20),  U (20)
      T  = 0.0
      DO 10  J =  1, N
10     T = T + U(J) * ( 1./(X(J) + B) -1. / B )
      RETURN
      END


      FUNCTION W ( N, SAM)
      DIMENSION SAM (20)
      W = 0.0
      DO 10  J =  1, N
10      W  = W  + SAM(J)  **2
      RETURN
      END
```

APPENDIX VI

SAMPLE CALCULATIONS

DATA FOR SINGLE SENIOR 4

INPUT DATA

X(J)DATA	PR(J)DATA	XN(J)DATA	PRN(J)DATA
10.000	0.	-10.000	0.500
20.000	0.450	-20.000	0.500
40.000	0.510	-40.000	0.530
100.000	0.550	-100.000	0.570
200.000	0.580	-200.000	0.610
400.000	0.630	-400.000	0.700

CALCULATIONS BASED ON PART I ONLY

X(J)DATA	L(J)DATA	XN(J)DATA	LN(J)DATA
10.000	1.000	-10.000	-1.000
20.000	1.222	-20.000	-1.222
40.000	1.174	-40.000	-1.324
100.000	1.083	-100.000	-1.436
200.000	1.040	-200.000	-1.627
400.000	0.955	-400.000	-2.229

JA = 1	U(J) =	1.222	UNEW(J) =	1.111
JJ = 1	UN(JJ) =	-1.000	UNNEW(JJ) =	-0.909
OJ = 2	UN(J) =	-1.222	UNNEW(J) =	-1.111
JA = 2	U(J) =	1.068	UNEW(J) =	1.089
JJ = 2	UN(JJ) =	-1.111	UNNEW(JJ) =	-1.134
OJ = 3	UN(J) =	-1.204	UNNEW(J) =	-1.228
JA = 3	U(J) =	1.005	UNEW(J) =	1.151
JJ = 3	UN(JJ) =	-1.228	UNNEW(JJ) =	-1.407
OJ = 4	UN(J) =	-1.332	UNNEW(J) =	-1.526
JA = 4	U(J) =	1.105	UNEW(J) =	1.316
JJ = 4	UN(JJ) =	-1.526	UNNEW(JJ) =	-1.817
OJ = 5	UN(J) =	-1.729	UNNEW(J) =	-2.058
JA = 5	U(J) =	1.209	UNEW(J) =	1.633
JB = 6	U(J) =	1.209	UNEW(J) =	0.722
JC = 7	U(J) =	1.209	UNEW(J) =	1.677
JJ = 5	UN(JJ) =	-2.058	UNNEW(JJ) =	-2.855
OJ = 6	UN(J) =	-2.820	UNNEW(J) =	-3.913

STEP-BY-STEP Calculations

CALCULATED VALUES BY STEP-BY-STEP METHOD

P(J)DATA	U(J)DATA	PX(J)DATA	UN(J)DATA
10.000	1.000	-10.000	-0.909
20.000	1.111	-20.000	-1.134
40.000	1.089	-40.000	-1.407
100.000	1.151	-100.000	-1.817
200.000	1.316	-200.000	-2.855
400.000	1.677	-400.000	-3.913

CALCULATIONS BASED ON NEQ = 1

N	X CALCULATED
1	0.400160E 03
2	0.574663E 00

XDATA	UDATA	UCALC	DEVIATION
-400.000	-3.913	-4.496	0.58285
-200.000	-2.855	-0.398	-2.45739
-100.000	-1.817	-0.165	-1.65188
-40.000	-1.407	-0.061	-1.34684
-20.000	-1.134	-0.029	-1.10432
-10.000	-0.909	-0.015	-0.89455
0.	0.	0.	-0.
10.000	1.000	0.014	0.98582
20.000	1.111	0.028	1.08308
40.000	1.089	0.055	1.03457
100.000	1.151	0.128	1.02330
200.000	1.316	0.233	1.08292
400.000	1.677	0.398	1.27882

STANDARD DEVIATION = 1.29136

SNOLSE CALCULATIONS FOR 2 COEFFICIENTS

FOR THREE COEFFICIENTS: NO CONVERGENCE IN 40 ITERATIONS IN SNOLSE.

FOR FOUR COEFFICIENTS: INSTABILITY IN SNOLSE.